MATHEMATICS

STUDY GUIDE

Curriculum Planning and Development Division

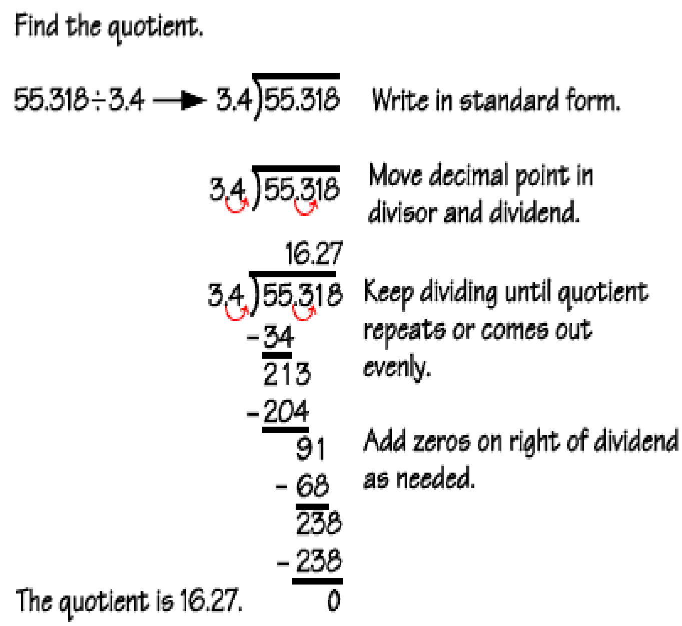
The booklet highlights some salient points for each topic in the CSEC Mathematics syllabus. At least one basic illustration/example accompanies each salient point. The booklet is meant to be used as a resource for “last minute” revision by students writing CSEC Mathematics.

Number Theory	
Basic Rules	
Points to Remember	**Illustration/ Example**
The sum of any number added to zero gives the same number	**The Additive Identity** $a + 0 = 0 + a = a$ 7 + 0 = 7 0 + 3.6 = 3.6
The product of any number multiplied by 1 gives the same number	**Multiplicative Identity** a × 1 = 1 × a = a **7** × 1 **=** 1 × 7 = **7**
Any number that is multiplied by zero gives a product of zero	a × 0 = 0 × a = 0 7 × 0 = 0 × 7 = 0
The sum (or difference) of 2 real numbers equals a real number	4 + 5 = 9 4 + (- 5) = -1 4.3 + 5.2 = 9.5
Zero divided by any number equals zero.	0/5 = 0 0/x = 0 $x \neq 0$
Any number that is divided by zero is undefined. The denominator of any fraction cannot have the value zero.	5/0 is undefined 0/0 is undefined x/0 is undefined $x \neq 0$
The Associative Law states The "Associative Laws" say that it doesn't matter how we group the numbers, the order in which numbers are added or multiplied does not affect their sum or product.	(a + b) + c **=** a + (b + c) (6 + 3) + 4 = 6 + (3 + 4) = 13 (a × b) × c = a × (b × c) (6 × 3) × 4 = 6 × (3 × 4) = 72
The Commutative Law states that in a set of numbers, multiplication must be applied before addition.	a + b + c = c + b + a = b + c + a 2 + 3 + 4 = 4 + 3 + 2= 3 + 4 + 2 = 9 a × b × c = c × b × a = b × c × a 2 × 3 × 4 = 4 × 3 × 2 = 3 × 4 × 2 = 24
BODMAS provides the key to solving mathematical problems **B - B**rackets first **O - O**rders (ie Powers and Square Roots, etc.) **DM- D**ivision and **M**ultiplication (left-to-right)	$7 + (6 \times 5^2 \times 3)$ Start inside Brackets, and then use "Orders" first = 7 + (6 × 25 + 3) Then Multiply = 7 + (150 + 3) Then Add = 7 + (153) Final operation is addition = 160 DONE!
When positive numbers are added together the result is positive	4 + 5 = 9
When two or more negative numbers are to be added, we simply add their values and get another negative number	- 4 – 5 = - 9

Number Theory		
Basic Rules		
Points to Remember	**Illustration/ Example**	
To find the difference between two numbers when one number is positive and one number is negative the result will be "+" if the larger value is positive or "–" negative if the larger number is negative.	20 – 10 = 10 -20 + 10 = -10	
When multiplying, two positive numbers multiplied together give a positive product; and a negative number multiplied by another negative number gives a positive product. Also, a negative number multiplied by a positive number gives a negative product	(+) × (+) = + (-) × (-) = + (+) × (-) = - (-) × (+) = -	e.g. 8 × 5 = 40 -8 × -5 = 40 8 × -5 = -40 -8 × 5 = -40

Number Theory		
Positive and Negative Numbers		
Points to Remember	**Illustration/ Example**	
The rules for division of directed numbers are similar to multiplication of directed numbers. **Use manipulatives- counters (yellow and red)**	(+) ÷ (+) = + (-) ÷ (-) = + (+) ÷ (-) = - (-) ÷ (+) = -	e.g. 10 ÷ 5 = 2 -10 ÷ -5 = 2 10 ÷ -5 = -2 -10 ÷ 5 = -2
There are different type of numbers: **Natural Numbers** - The whole numbers from 1 upwards **Integers**- The whole numbers, {1,2,3,...} negative whole numbers {..., -3,-2,-1} and zero {0}. **Rational Numbers**- The numbers you can make by dividing one integer by another (but not dividing by zero). In other words, fractions. **Irrational Number** – Cannot be written as a ratio of two numbers **Real Numbers** - All Rational and Irrational numbers. They can also be positive, negative or zero.	Natural Numbers (N) : {1,2,3,...} Integers (Z) : {..., -3, -2, -1, 0, 1, 2, 3, ...} Rational Numbers (Q) :. 3/2 (=1.5), 8/4 (=2), 136/100 (=1.36), -1/1000 (=-0.001) Irrational Number : π, 3.142 (cannot be written as a fraction) Real Numbers (R): 1.5, -12.3, 99, $\sqrt{2}$, π	

Number Theory	
Decimals – Rounding	
Points to Remember	**Illustration/ Example**
Rounding up a decimal means increasing the terminating digit by a value of 1 and drop off the digits to the right. Round down if the number to the right of our terminating decimal place is four or less (4,3,2,1,0)	5.47 to the tenths place, it can be can be rounded up to 5.5 6.734 to the hundredths place, it can be rounded down to 6.73

Number Theory	
Operations with Decimals	
Points to Remember	**Illustration/ Example**
Find the product of 3.77 x 2.8 =? 1. Line up the numbers on the right, 2. multiply each digit in the top number by each digit in the bottom number (like whole numbers), 3. add the products, 4. and mark off decimal places equal to the sum of the decimal places in the numbers being multiplied.	Find the product of 3.77 × 2.8 3.77 (2 decimal places) <u> 2.8 </u> (1 decimal place) 754 <u>3016</u> 10.556 (3 decimal places)
When dividing, if the divisor has a decimal in it, make it a whole number by moving the decimal point to the appropriate number of places to the right. If the decimal point is shifted to the right in the divisor, also do this for the dividend.	
Fractions can always be written as decimals.	For example: $\frac{2}{5} = 0.4$ $\frac{1}{2} = 0.5$ $\frac{3}{4} = 0.75$ $\frac{1}{4} = 0.25$ $\frac{3}{5} = 0.6$ $\frac{3}{4} = 0.75$

Number Theory

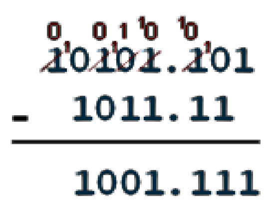
Significant figures

Points to Remember	Illustration/ Example
The rules for significant figures: 1) ALL non-zero numbers (1,2,3,4,5,6,7,8,9) are ALWAYS significant. 2) ALL zeroes between non-zero numbers are ALWAYS significant. 3) ALL zeroes which are SIMULTANEOUSLY to the right of the decimal point AND at the end of the number are ALWAYS significant. 4) ALL zeroes which are to the left of a written decimal point and are in a number >= 10 are ALWAYS significant. A helpful way to check rules 3 and 4 is to write the number in scientific notation. If you can/must get rid of the zeroes, then they are NOT significant.	See table below

Number	Number of Significant Figures	Rule(s)
48,923	5	1
3.967	4	1
900.06	5	1,2,4
0.0004 (= 4 E-4)	1	1,4
8.1000	5	1,3
501.040	6	1,2,3,4
3,000,000 (= 3 E+6)	1	1
10.0 (= 1.00 E+1)	3	1,3,4

Number Theory

Binary Numbers

Points to Remember	Illustration/ Example
Each digit "1" in a binary number represents a power of two, and each "0" represents zero.	0001 is 2 to the zero power, or 1 0010 is 2 to the 1st power, or 2 0100 is 2 to the 2nd power, or 4 1000 is 2 to the 3rd power, or 8
Binary numbers can be added	10001 +11101 101110
Binary numbers can be subtracted	

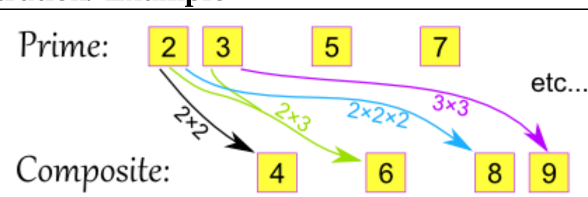
Number Theory

Computation – Fractions

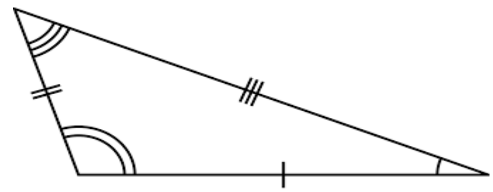
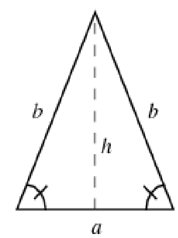
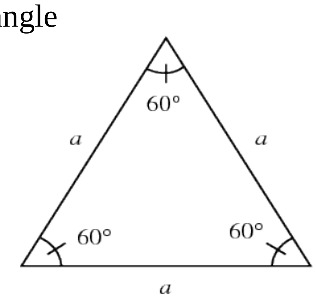
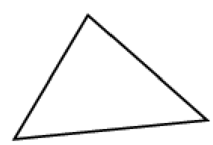
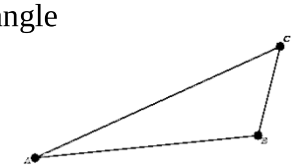
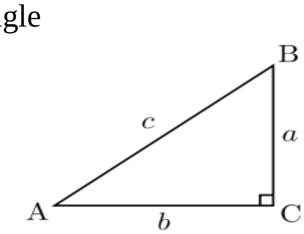
Points to Remember	Illustration/ Example
When the numerator stays the same, and the denominator increases, the value of the fraction decreases	$\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ $\frac{3}{4}, \frac{3}{5}, \frac{3}{6}\ \frac{3}{7}$
When the denominator stays the same, and the numerator increases, the value of the fraction increases.	$\frac{7}{2}, \frac{8}{2}, \frac{9}{2}$
Equivalent fractions are fractions that may look different, but are equal to each other.	$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}\ \frac{4}{8}$
Equivalent fractions can be generated by multiplying or dividing both the numerator and denominator by the same number.	$\frac{1}{2} = \frac{1x2}{2x2} = \frac{2}{4}$ $\frac{3}{5} = \frac{3x2}{5x2} = \frac{6}{10}$
Fractions can be simplified when the numerator and denominator have a common factor in them	$\frac{6}{10} = \frac{3x2}{5x2} = \frac{3}{5}$
Fractions with different denominators, can be converted to a set of fractions that have the same denominator	$\frac{3}{4}, \frac{2}{3}$ is the same as $\frac{9}{12}, \frac{8}{12}$
Addition and subtraction of fractions are similar to adding and subtracting whole numbers if the fractions being added or subtracted have the same denominator	$\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$
When multiplying fractions, multiply the numerators together and then multiply the denominators together and simplify the results.	$\frac{5}{6} \times \frac{2}{3} = \frac{10}{18} = \frac{5}{9}$

Number Theory

Prime Numbers

Points to Remember	Illustration/ Example
A prime number is a number that has only two factors: itself and 1e.g. 5 can only be divided evenly by 1 or 5, so it is a prime number. Numbers that are not prime numbers are referred to as composite numbers	

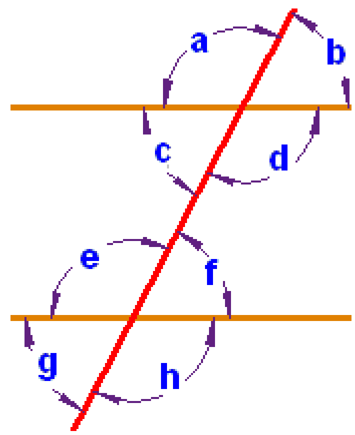
Number Theory	
Computation of Decimals, Fractions and Percentages	
Points to Remember	**Illustration/ Example**
Percent means "per one hundred"	20 % = 20 per 100
To convert from percent to decimal, divide the percent by 100	10 % = $\frac{10}{100}$ = 0.1 67.5% = $\frac{67.5}{100}$ = 0.675
To convert from decimal to percent, multiply the decimal by 100	0.10 as a percentage is 0.10 × 100 = 10% 0.675 is 0.675 × 100 = 67.5%
To convert from percentages to fractions, divide the percent by 100 to get a fraction and then simplify the fraction	12% = $\frac{12}{100}$ = $\frac{12 \div 4}{100 \div 4}$ = $\frac{3}{25}$
To convert from fractions to percentages, convert the fraction to a decimal by dividing the numerator by the denominator and then convert the decimal to a percent by multiplying by 100.	$\frac{3}{25}$ = 0.12 0.12 as a percentage is 0.12 × 100 = 12%

Triangles	
Classification of Triangles	
Points to Remember	**Illustration/ Example**
Triangles can be classified according to lengths of their sides to fit into three categories:- Scalene: **No** equal sides ;**No** equal angles Isosceles: **Two** equal sides ; **Two** equal angles Equilateral Triangle: **Three** equal sides ; **Three** equal 60° angles	Scalene Triangle  Isosceles triangle  Equilateral Triangle 
Triangles can be classified according to angles:- Acute- All three angles are acute angles. Obtuse- An obtuse triangle is a triangle in which one of the angles is an obtuse angle. Right angle- A triangle that has a right angle (90°)	acute angle triangle  obtuse angle triangle  right angle triangle 

<table>
<tr><th colspan="2">Angles formed by a Transversal Crossing two Parallel Lines</th></tr>
<tr>
<td>Vertical Angles are the angles opposite each other when two lines cross.
Vertically opposite angles are equal
a = d f = g
b = c e = h

The angles in matching corners are called Corresponding Angles.
Corresponding Angles are equal
a = e c = g
b = f d = h

The pairs of angles on opposite sides of the transversal but inside the two lines are called Alternate Angles.
Alternate Angles are equal
d = e c = f

The pairs of angles on one side of the transversal but inside the two lines are called Consecutive Interior Angles.
Consecutive Interior Angles are supplementary (add up to 180°)
d + f c + e</td>
<td>Illustration of all angles mentioned on a single diagram. The transversal crosses two Parallel Lines

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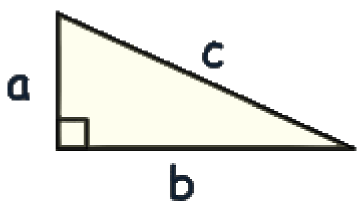
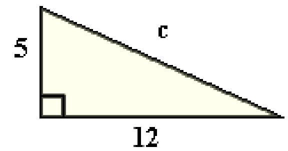
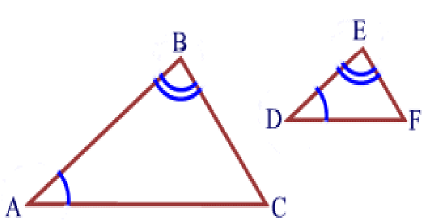
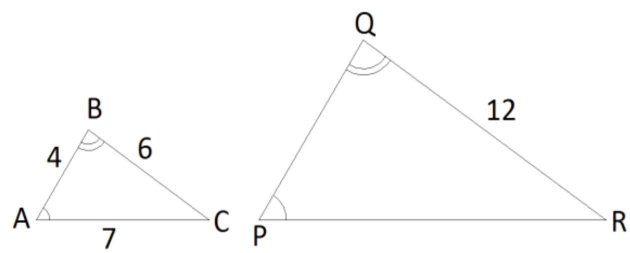
Triangles	
Pythagoras' Theorem	
Points to Remember	**Illustration/ Example**
Pythagoras' Theorem states that the square of the hypotenuse is equal to the sum of the squares on the other two sides 	$c^2 = a^2 + b^2$ The Hypotenuse is c Find c  $c^2 = 5^2 + 12^2$ $= 25 + 144$ $= 169$ $c = \sqrt{169}$ $= 13$ units

Triangles	
Similar Triangles & Congruent Triangles	
Points to Remember	**Illustration/ Example**
Definition: Triangles are similar if they have the same shape, but can be different sizes. (They are still similar even if one is rotated, or one is a mirror image of the other). There are three accepted methods of proving that triangles are similar: If two angles of one triangle are equal to two angles of another triangle, the triangles are similar.  If angle A = angle D and angle B = angle E Then ΔABC is similar to ΔDEF	Show that the two triangles given beside are similar and calculate the lengths of sides PQ and PR.  **<u>Solution:</u>** ***∠A = ∠P*** and ***∠B = ∠Q, ∠C = ∠R***(because ∠C = 180 - ∠A - ∠B and ∠R = 180 - ∠P - ∠Q) Therefore, the two triangles ΔABC and ΔPQR are similar.

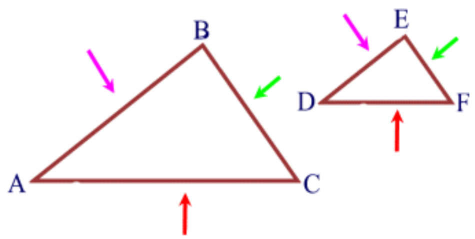
Triangles

Similar Triangles & Congruent Triangles

Points to Remember	**Illustration/ Example**

Points to Remember

1) If the three sets of corresponding sides of two triangles are in proportion, the triangles are similar.

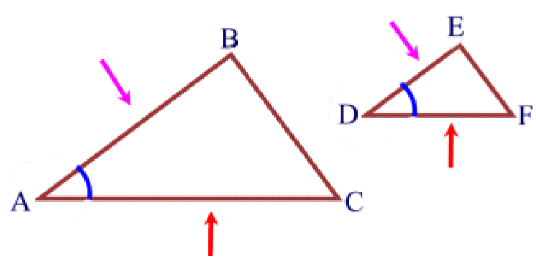


If

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

Then ΔABC is similar to ΔDEF

2) If an angle of one triangle is equal to the corresponding angle of another triangle and the lengths of the sides including these angles are in proportion, the triangles are similar.



If angle A = angle D and

$$\frac{AB}{DE} = \frac{AC}{DF}$$

Then ΔABC is similar to ΔDEF

Illustration/ Example

Consequently:

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ implies $\frac{AB}{PQ} = \frac{BC}{QR}$

Substituting known lengths give: $\frac{4}{PQ} = \frac{6}{12}$ or 6PQ = 4 × 12

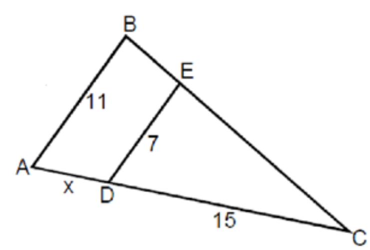
Therefore PQ = $\frac{12 \times 4}{6}$ = 8

Also, $\frac{BC}{QR} = \frac{AC}{PR}$

Substituting known lengths give: $\frac{6}{12} = \frac{7}{PR}$ or 6PR = 12 × 7

Therefore PR = $\frac{12 \times 7}{6}$ = 14

Find the length ***AD (x)***

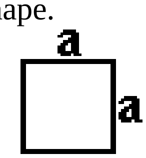
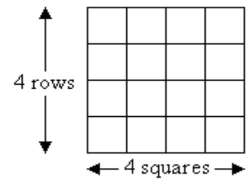
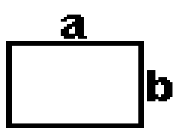
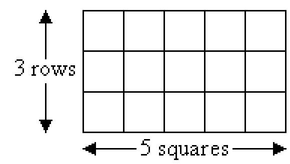
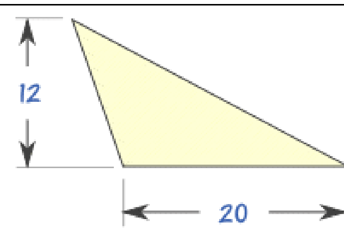
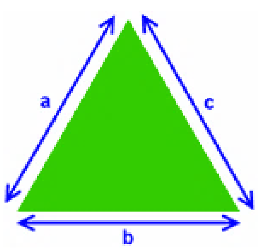


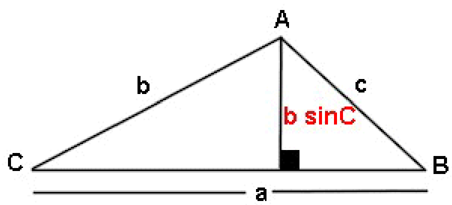
The two triangles ΔABC and ΔCDE appear to be similar since AB || DE and they have the same apex angle C.
It appears that one triangle is a scaled version of the other. However, we need to prove this mathematically.

AB || DE, CD || AC and BC || EC
∠BAC = ∠EDC and ∠ABC = ∠DEC

Considering the above and the common angle ***C***, we may conclude that the two triangles ΔABC and ΔCDE are similar.

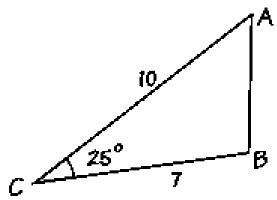
Triangles	
Similar Triangles & Congruent Triangles	
Points to Remember	**Illustration/ Example**
	Therefore: $\frac{DE}{AB} = \frac{CD}{CA}$ $\frac{7}{11} = \frac{15}{CA}$ $7CA = 11 \times 15$ $CA = \frac{11 \times 15}{7}$ CA = 23.57 x = CA – CD = 23.57 – 15 = 8.57

Mensuration	
Areas & Perimeters	
Points to Remember	**Illustration/ Example**
The area of a shape is the total number of square units that fill the shape.  **Area** of Square= a^2 **Perimeter** of Square= a+ a + a + a a = length of side	Find the area and perimeter of a square that has a side-length of 4 cm  Area of Square = a × a= a^2 = 4 × 4= 4^2 = 16 cm^2 Perimeter of Square = 4 + 4 + 4 + 4 = 16 cm
 **a** represents the length; **b** represents the width **Area** of Rectangle = a × b **Perimeter** of Rectangle = a+ a + b + b = 2(a+b)	Find the area of a rectangle of length 5cm, width 3cm  Area of Rectangle = 5 cm x 3cm = 15 cm^2 Perimeter of Rectangle = 5 + 5 + 3 + 3 = 2(5+3) = 16cm
The area of a triangle is : $\frac{1}{2}$ x b x h b is the base h is the height	 Area = $\frac{1}{2}$ x b x h = $\frac{1}{2}$ x 20units x 12units = 120 $units^2$
Area of triangle using "Heron's Formula"- given all three sides:  **Step 1:** Calculate **"s"** (half of the triangle's perimeter): $s = \frac{a+b+c}{2}$ **Step 2:** Then calculate the **Area**:	Example: What is the area and perimeter of a triangle with sides 3cm, 4cm and 5cm respectively? Step 1: s = $\frac{3+4+5}{2}$ = $\frac{12}{2}$ = 6 Step 2 **:** **Area** of triangle = $\sqrt{6(6-3)(6-4)(6-5)}$ = $\sqrt{6\ (3)(2)(1)}$ = 6cm^2 **Perimeter** of triangle = a + b + c = 3 + 4 + 5 = 12cm

<table>
<tr><td colspan="2">Mensuration</td></tr>
<tr><td colspan="2">Areas & Perimeters</td></tr>
<tr><td>Points to Remember</td><td>Illustration/ Example</td></tr>
<tr><td>$A = \sqrt{s(s-a)(s-b)(s-c)}$</td><td></td></tr>
<tr><td>Area of triangle, given two sides and the angle between them


Either Area = ½ ab sin C
or Area = ½ bc sin A
or Area = ½ ac sin B

Or in general,

Area = ½ × side 1 × side 2 × sine of the included angle</td><td>

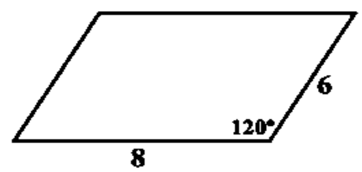
First of all we must decide what we know. We know angle C = 25º, and sides a = 7 and b = 10.

Start with:
Area= (½)ab sin C

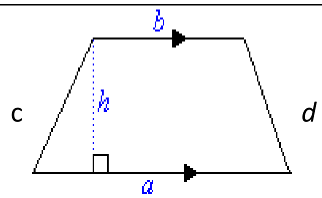
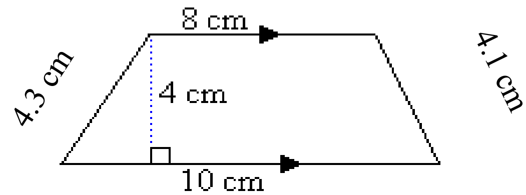
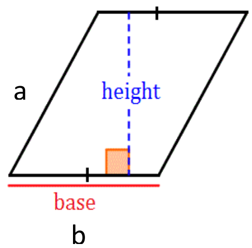
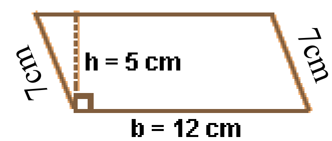
Put in the values we know: Area=½ × 7 × 10 × sin(25º)

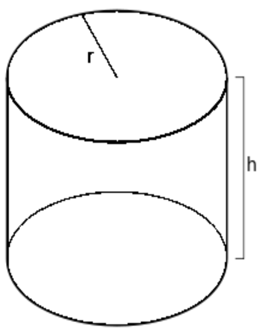
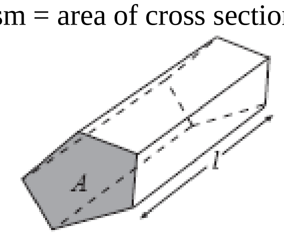
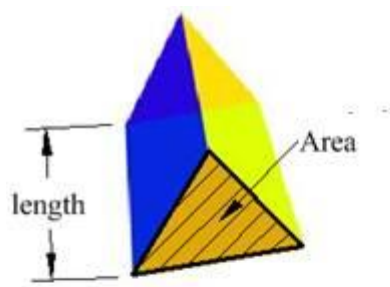
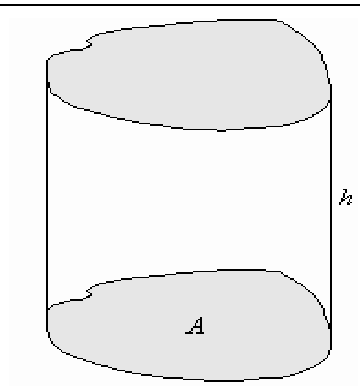
Do some calculator work:
Area = 35 × 0.4226 =14.8 units^2 (1dp)</td></tr>
<tr><td>Area of Parallelogram, given two sides and an angle

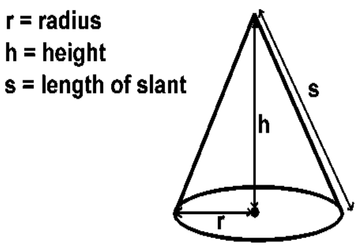
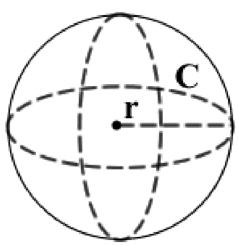
The diagonal of a parallelogram divides the parallelogram into two congruent triangles. Consequently, the area of a parallelogram can be thought of as doubling the area of one of the triangles formed by a diagonal. This gives the trig area formula for a parallelogram:

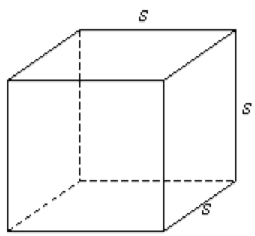
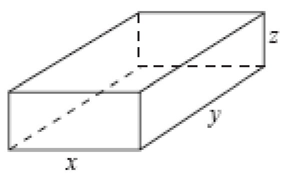
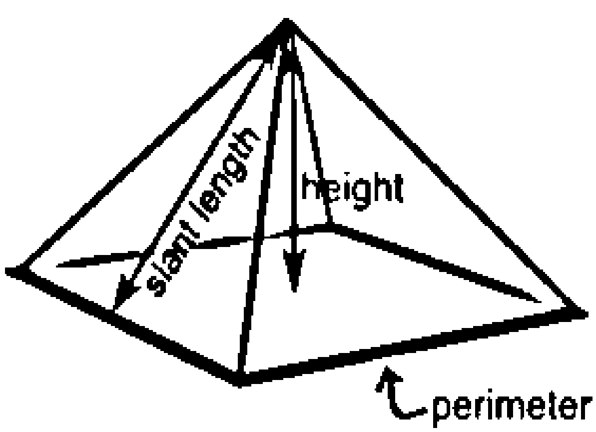
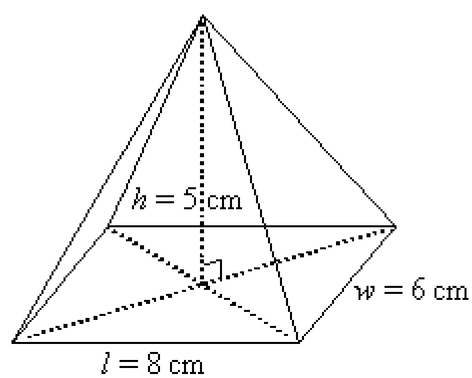
Either Area = ab sin C
or Area = bc sin A
or Area = ac sin B</td><td>Find the area of the parallelogram:


Area = ab sin C
= (8)(6)sin 120^o
= 41.569 =41.57 square units</td></tr>
</table>

Mensuration	
Areas & Perimeters	
Points to Remember	**Illustration/ Example**
 Area of Trapezium = ½(a+b) × h = ½(sum of parallel sides) × h *h* = vertical height **Perimeter** = *a* + *b* + *c* + *d*	Find the area of the trapezium  A = ½ (a+b) × h = ½(10 + 8) × 4 =½ × (18) × 4 = 36 cm^2 **Perimeter** = a + b + c + d = 10 + 8 + 4.3 + 4.1 = 26.4 cm
 Area of Parallelogram = base × height b = base h= vertical height	Find the area of a parallelogram with a base of 12 centimeters and a height of 5 centimeters.  **Area** of parallelogram= b × h = 12cm × 5cm = 60 cm^2 **Perimeter** of parallelogram **=** a + b + a + b = 2 (a + b) = 12cm + 7cm + 12cm + 7cm = 38cm

Mensuration	
Surface Area and Volumes	
Points to Remember	**Illustration/ Example**
Volume of Cylinder = Area of Cross Section x Height = πr^2h **Surface Area** of Cylinder = $2\pi r^2 + 2\pi rh = 2\pi r(r + h)$ 	Find the volume and total surface area of a cylinder with a base radius of 5 cm and a height of 7 cm. **Volume** = $\pi r^2h = \frac{22}{7} \times 5^2 \times 7 = 22 \times 25\ cm^3 = 550 cm^3$ Conversion to **Litres**: $1000\ cm^3 = 1L$ $550\ cm^3 = \frac{550}{1000} L = 0.55\ L$ **Surface Area** = $2\pi(5)(7) + 2\pi(5)^2 = 70\pi + 50\pi$ $= 120\pi\ cm^2$ $\approx 376.99\ cm^2$
* A prism is a three-dimensional shape which has the same shape and size of cross-section along the entire length i.e. a uniform cross-section Prism- Since a cylinder is closely related to a prism, the formulas for their surface areas are related Volume of Prism = area of cross section × length = ***A l*** 	Example: What is the volume of a prism whose ends have an area of 25 m^2 and which is 12 m long  Answer: Volume = $25\ m^2 \times 12\ m = 300\ m^3$
 **Volume** of irregular prism = **A**h **Surface Area** of irregular prism = 2**A** + *(perimeter of base* × h)	

Mensuration	
Surface Area and Volumes	
Points to Remember	**Illustration/ Example**
 **Volume of cone** = $\frac{1}{3}\pi r^2 h$ **The slant** of a right circle cone can be figured out using the Pythagorean Theorem if you have the height and the radius. **Surface area** $= \pi rs + \pi r^2$	What is the volume and surface area of a cone with radius 4 cm and slant 8 cm? **Slant Height using Pythagoras' Theorem:** **h=** $\sqrt{s^2 - r^2}$ $= \sqrt{8^2 - 4^2}$ $= \sqrt{64 - 16}$ $= \sqrt{48}$ $= 6.928 \approx 6.93$ **Volume of cone** $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 4^2 \times 6.93 = 116.05 \text{ cm}^3$ **Surface area** $= \pi rs + \pi r^2$ $= (3.14 \times 4 \times 8) + (3.14 \times 4^2)$ $= 100.48 + 50.24$ $= 150.72 \text{ cm}^2$
 **Volume** of Sphere: **V =** $\frac{4}{3}\pi r^3$ **Surface area** of a sphere: **A =** $4\pi r^2$	Find the volume and surface area and of a sphere with radius 2 cm **Volume** of Sphere $= \frac{4}{3}\pi r^3$ $= \frac{4}{3} \times 3.14 \times 2^3$ $= \frac{100.48}{3}$ $= 33.49 \text{ cm}^3$ **Surface Area** of Sphere $= 4\pi r^2$ $= 4 \times 3.14 \times 2^2$ $= 50.24 \text{ cm}^2$

Mensuration	
Surface Area and Volumes	
Points to Remember	**Illustration/ Example**
 **Volume** of cube = $\mathbf{s^3}$ **Surface Area** of cube $= s^2 + s^2 + s^2 + s^2 + s^2 + s^2$ $= \mathbf{6\ s^2}$	Find the volume and surface area of a cube with a side of length 3 cm **Volume** of cube $= s \times s \times s = s^3 = 3 \times 3 \times 3 = 27\ cm^3$ **Surface Area** of cube $= s^2 + s^2 + s^2 + s^2 + s^2 + s^2$ $= 6\ s^2 = 6(3)^2 = 6 \times 9 = 54\ cm^2$
 **Volume** of cuboid = length x breadth x height $= \boldsymbol{xyz}$ **Surface** area $= xy + xz + yz + xy + xz + yz$ $= 2\mathbf{xy} + 2\mathbf{xz} + 2\mathbf{yz}$ $= 2(\mathbf{xy} + \mathbf{xz} + \mathbf{yz})$	Find the volume and surface area of a cuboid with length 10cm, breadth 5cm and height 4cm. **Volume** of cuboid = length × breadth × height $= 10 \times 5 \times 4$ $= 200 cm^3$ **Surface** Area of cuboid $= 2xy + 2xz + 2yz$ $= 2(10)(5) + 2(10)(4) + 2(5)(4)$ $= 100 + 80 + 40$ $= 220\ cm^2$
 **The Volume of a Pyramid** $= \frac{1}{3} \times$ [Base Area] × Height	**Find the volume of a rectangular-based pyramid whose base is 8 cm by 6 cm and height is 5 cm.**  **Solution:** $V = \frac{1}{3} \times$ [Base Area] × Height

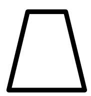
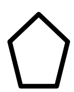
Mensuration	
Surface Area and Volumes	
Points to Remember	**Illustration/ Example**
	$= \frac{1}{3} \times [8 \times 6] \times 5$ $= 80 \text{ cm}^3$

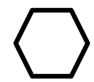
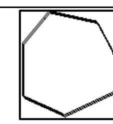
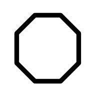
Geometry	
Sum of all interior angles of a regular polygon	
Points to Remember	**Illustration/ Example**
The sum of interior angles of a polygon having n sides is (2n – 4) right angles = (2n-4) x 90 . Each interior angle of the polygon = (2n – 4)/n right angles. e.g. What is the sum of the interior angles of a triangle	Find the sum of all interior angkes in i) Pentagon ii) Hexagon iii) Heptagon iv) Octagon

Geometry

Sum of all interior angles of any polygon

Illustration/ Example

Name	Figure	No. of Sides	Sum of interior angles (2n - 4) right angles
Triangle		3	(2n - 4) right angles = (2 × 3 - 4) × 90° = (6 - 4) × 90° = 2 × 90° = 180°
Quadrilateral		4	(2n - 4) right angles = (2 × 4 - 4) × 90° = (8 - 4) × 90° = 4 × 90° = 360°
Pentagon		5	(2n - 4) right angles = (2 × 5 - 4) × 90° = (10 - 4) × 90° = 6 × 90° = 540°

Name	Figure	No. of Sides	Sum of interior angles (2n - 4) right angles
Hexagon		6	(2n - 4) right angles = (2 × 6 - 4) × 90° = (12 - 4) × 90° = 8 × 90° = 720°
Heptagon		7	(2n - 4) right angles = (2 × 7 - 4) × 90° = (14 - 4) × 90° = 10 × 90° = 900°
Octagon		8	(2n - 4) right angles = (2 × 8 - 4) × 90° = (16 - 4) × 90° = 12 × 90° = 1080°

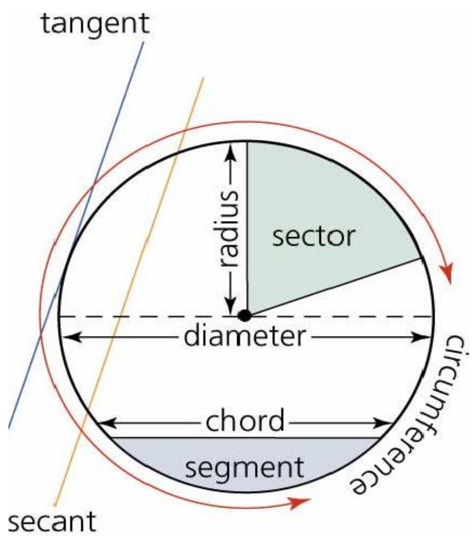
Geometry	
Sum of all exterior angles of any polygon	
Points to Remember	**Illustration/ Example**
Sum of all exterior angles of any polygon = 360° e.g. Find the sum of the exterior angles of: a) a pentagon Answer: 360^0 b) a decagon Answer: 360^0 c) a 15 sided polygon Answer: 360^0 d) a 7 sided polygon Answer: 360^0	Find the measure of each exterior angle of a regular hexagon A hexagon has 6 sides, so $n = 6$ Substitute in the formula Each Exterior angle = $\frac{360}{n}$ $=\frac{360}{60}$ $=60^o$ The measure of **each** exterior angle of a regular polygon is 45°. How many sides does the polygon have? Set the formula equal to 45^o. Cross multiply and solve for n $\frac{360}{n} = 45$ $45n = 360$ $n = \frac{360}{45} = 8$

Geometry

Circle Geometry

Points to Remember

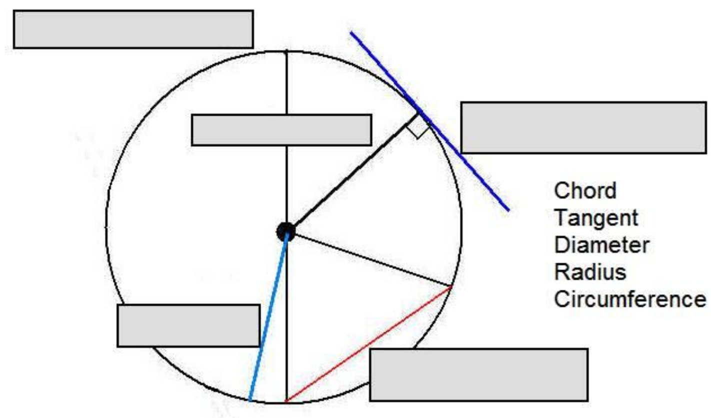
Parts of a Circle



- **Arc** — a portion of the circumference of a circle.
- **Chord** — a straight line joining the ends of an arc.
- **Circumference** — the perimeter or boundary line of a circle.
- **Radius** (r) — any straight line from the centre of the circle to a point on the circumference.
- **Diameter** — a special chord that passes through the centre of the circle. A diameter is a straight line segment from one point on the circumference to another point on the circumference that passes through the centre of the circle.
- **Segment** — part of the circle that is cut off by a chord. A chord divides a circle into two segments.
- **Tangent** — a straight line that makes contact with a circle at only one point on the circumference

Illustration/ Example

Insert the parts of the circle below:



Geometry

Circle Geometry

<table>
<tr><th>Points to Remember</th><th>Illustration/ Example</th></tr>
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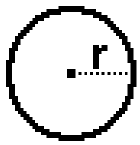

Area = $\pi \times r^2$

Circumference = $2 \times \pi \times r = D \times \pi$
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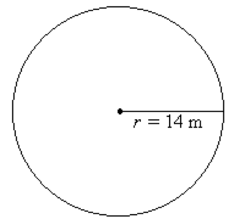


Area of Circle=

$= \pi \times r^2$

$= \frac{22}{7} \times 14\text{m} \times 14\text{m}$

$= \frac{22}{7} \times 196 \text{ m}^2 = 616 \text{ m}^2$

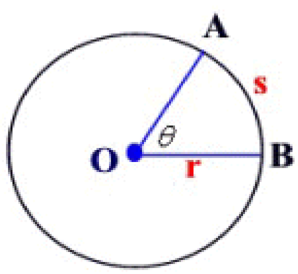
Perimeter (Circumference) of circle

$= 2 \times \pi \times r$

$= 2 \times \frac{22}{7} \times 14 = 87.976 \text{ cm} = 87.98 \text{ cm}$ to (2 dp)
</td></tr>
<tr><td>
Area of Sector AOB $= \pi \times r^2 \times \frac{\theta}{360}$

Length of Arc AB $= 2\pi\, r \times \frac{\theta}{360}$

Perimeter = BO + OA + arc AB



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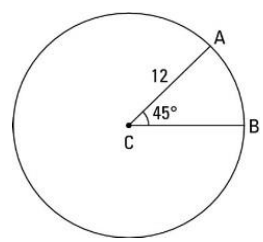


Area of Sector = $\pi \times r^2 \times \frac{\theta}{360}$

$= \frac{22}{7} \times 12 \times 12 \times \frac{45}{360} = 56.55 \text{ units}^2$

Arc length AB = $2\pi r \times \frac{\theta}{360}$

$= 2 \times \frac{22}{7} \times 12 \times \frac{45}{360}$

= 9.428

= 9.43 units (2dp)

Perimeter of sector ABC

= BC + CA + Arc length AB

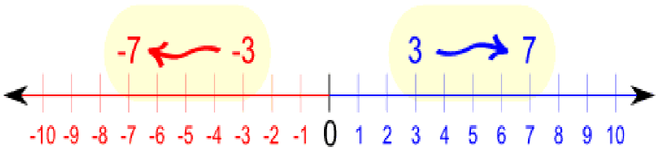
= 12 + 12 + 9.43 = 33.43 units
</td></tr>
</table>

Algebra	
Simplifying algebraic expressions	
Points to Remember	**Illustration/ Example**
Algebraic expressions are the phrases used in algebra to combine one or more variables, constants and the operational (+ - x /) symbols. Algebraic expressions don't have an equals = sign. Letters are used to represent the variables or the constants	1) Nine increased by a number x 9 + x 2) Fourteen decreased by a number p 14 – p 3) Seven less than a number t t – 7 4) The product of nine and a number, decreased by six 9m – 6 5) Three times a number, increased by seventeen 3a + 17 6) Thirty-two divided by a number y 32 ÷ y 7) Five more than twice a number 2n + 5 8) Thirty divided by seven times a number 30 ÷ 7n

Algebra	
Substitution	
Points to Remember	**Illustration/ Example**
In Algebra "Substitution" means putting numbers where the letters are	1) If x = 5, then what is $\mathbf{\frac{10}{x} + 4}$ $\frac{10}{5} + 4 = 2 + 4 = 6$ 2) If x = 3 and y = 4, then what is $\mathbf{x^2 + xy}$ $3^2 + 3 \times 4 = 9 + 12 = 21$ 3) If x = **−2**, then what is $\mathbf{1 - x + x^2}$ $\mathbf{1 - (-2) + (-2)^2} = 1 + 2 + 4 = 7$

Algebra	
Binary Operations	
Points to Remember	**Illustration/ Example**
A binary operation is an operation that applies to two numbers, quantities or expressions e.g. a*b = 3a + 2b **Commutative Law** Let * be a binary operation. * is said to be commutative if , a * b = b * a **Associative Law** Let * be a binary operation. * is said to be an associative if , a * (b * c) = (a * b) * c	An operation * is defined by a * b = 3a + b. Determine: i) 2*4 ii) 4*2 iii) (2*4)*1 iv) 2* (4*1) v) Is * associative? vi) Is * communicative? i) 2*4 = 3(2) + 4 = 10 ii) 4*2 = 3(4) + 2 = 14 iii) (2*4)*1= 10*1= 3(10) + 1 = 31 iv) 2* (4*1)= 2 * [3(4)+1]= 2*13= 3(2) +13=19 v) Since (2*4)*1 ≠ 2* (4*1), * is not associative. That is (a*b)*c, ≠ a* (b*c) vi) Since 2*4 *≠ 4*2, * is not commutative. That is a*b ≠ b*a

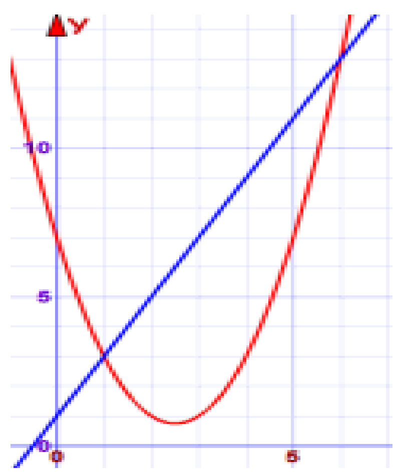
Algebra	
Solving Linear Equations	
Points to Remember	**Illustration/ Example**
An equation shows the link between two expressions	1)Solve 2x + 6 = 10 2x + 6 = 10 2x = 10-6 2x = 4 x $= \frac{4}{2}$ x = 2 2) Solve 5x – 6 = 3x – 8 5x – 6 = 3x – 8 5x – 3x = -8 + 6 2x = -2 x $= \frac{-2}{2}$ x = -1

Algebra	
Linear Inequalities	
Points to Remember	**Illustration/ Example**
*Solving linear inequalities is almost exactly like solving linear equations * When we multiply or divide by a **negative number,** we must **reverse** the inequality Why? For example, from 3 to 7 is **an increase,** but from -3 to -7 is **a decrease**  **-7 < -3** **7 > 3** The inequality sign reverses (from < to >)	1) Solve $x + 3 < 0$ $x < -3$ 2) $3y < 15$ $y < \frac{15}{3}$ $y < 5$ 3) $(x-3)/2 < -5$ $(x-3) < -10$ $x < -7$ 4) $-2y < -8$ divide both sides by -2 ... and **reverse the inequality** $y > \frac{-8}{-2}$ $y > 4$

Algebra	
Changing the Subject of a Formula	
Points to Remember	**Illustration/ Example**
Formula means Relationship between two or more variables Example: y = x + 5 where x and y are variables. **Subject Of A Formula means** The variable on its own, usually on the left hand side. Example: y is the subject of the formula y = x + 5 **Changing The Subject Of A Formula means** Rearrange the formula so that a different variable is on its own. Example: Making x the subject of the formula y = x + 5 gives x = y - 5	Make x the subject of the formula $y = x + 5$ $x + 5 = y$ $x = y - 5$ Make x the subject of the formula $y = 3x - 6$ Switch sides $3x - 6 = y$ $3x = y + 6$ $x = \frac{y+6}{3}$ Make x the subject of the formula $y = 2(x + 5)$ switch sides $2(x + 5) = y$ Multiply out brackets $2(x) + 2(5) = y$

Algebra	
Changing the Subject of a Formula	
Points to Remember	**Illustration/ Example**
	$2x + 10 = y$ $2x = y - 10$ $x = \frac{y-10}{2}$ Make x the subject of the formula $y = \frac{x}{2} + 5$ multiply everything on both sides by 2 $\frac{x}{2} + 5 = y$ $2(\frac{x}{2}) + 2(5) = 2(y)$ $x + 10 = 2y$ $x = 2y - 10$ Make v the subject of the formula $E = \frac{1}{2} mv^2$ Switch sides $\frac{1}{2} mv^2 = E$ Multiply everything across by 2 $2(\frac{1}{2} mv^2) = 2E$ $mv^2 = 2E$ $v^2 = \frac{2E}{m}$ $v = \sqrt{\frac{2E}{m}}$ Make x the subject of the formula $\frac{xw}{t} + p = y$ Multiply everything on both sides by t $t(\frac{xw}{t}) + tp = ty$ $xw + tp = ty$ $xw = ty - tp$ $x = \frac{ty-tp}{w}$

Algebra	
Solving Simultaneous Equations (both Linear)	
Points to Remember	**Illustration/ Example**
*Simultaneous means "at the same time" *There are two methods used for solving systems of equations: the elimination method and substitution method * The elimination method is most useful when one variable from both equations has the same coefficient in both equations, or the coefficients are multiples of one another. * To use the substitution method, two conditions must be met: there must be the same number of equations as variables; and one of the equations must be easily solved for one variable	Solve simultaneously using the **elimination method** : $3x - y = 1$ ….eq. (1) $2x + 3y = 8$ ….eq. (2) Eq.(1) x 3 … $9x - 3y = 3$ Eq. (2) x 1 … $\underline{2x + 3y = 8}$ Add both equations $11x = 11$ $x = \frac{11}{11}$ $x = 1$ Substitute x=1 into Eq. (1) $3(1) - y = 1$ $3 - 1 = y$ $2 = y$ Solution (1, 2) Using the **substitution method**: Make y the subject of the formula in Eq. (1) $3x - y = 1$ $3x - 1 = y$ Substitute $y = 3x - 1$ into Eq. (2) $2x + 3(3x - 1) = 8$ $2x + 9x - 3 = 8$ $11x - 3 = 8$ $11x = 8 + 3$ $11x = 11$ $x = \frac{11}{11}$ $x = 1$ Substitute x=1 into Eq. (1) $3(1) - y = 1$ $3 - 1 = y$ $2 = y$ Solution (1, 2)

Algebra	
Solving Simultaneous Equations (Linear and Quadratic)	
Points to Remember	**Illustration/ Example**
	Solve simultaneously: $2x + y = 7$ Eq. (1) $x^2 - xy = 6$ Eq. (2) From Eq.(1), $2x + y = 7$ $y = 7 - 2x$ Substituting this value of y into Eq. (2) $x^2 - x(7 - 2x) - 6 = 0$ $x^2 - 7x + 2x^2 - 6 = 0$ $3x^2 - 7x - 6 = 0$ $(3x + 2)(x - 3) = 0$ $x = \frac{-2}{3}$ or $x = 3$ Using Eq. 1, when $x = \frac{-2}{3}$ $y = 7 - 2\left(\frac{-2}{3}\right)$ $= 7 + \left(\frac{4}{3}\right)$ $= \frac{25}{3}$ When $x = 3$ $y = 7 - 2(3)$ $= 7 - 6$ $= 1$ Solutions: $\left(\frac{-2}{3}, \frac{25}{3}\right)$ or $(3, 1)$
Solving a pair of equations in two variables (linear and quadratic) Use graphs to find solutions to simultaneous equations $y = x^2 - 5x + 7$ … eq. (1) $y = 2x + 1$ … eq. (2) Set them equal to each other $x^2 - 5x + 7 = 2x + 1$ $x^2 - 5x - 2x + 7 - 1 = 0$ $x^2 - 7x + 6 = 0$ $(x - 1)(x - 6) = 0$ $x = 1$ and $x = 6$ Substitute into eq. (2) When $x = 1$; $y = 2(1) + 1 = 3$ $x = 6$; $y = 2(6) + 1 = 13$ Solutions (1, 3) and (6, 13)	

Algebra	
Indices	
Points to Remember	**Illustration/ Example**
The laws of indices are used to simplify expressions and numbers with indices e.g. $2^5 = 2 \times 2 \times 2 \times 2 \times 2$ **Laws of Indices** $a^0 = 1$ $a^m \times a^n = a^{m+n}$ $a^m \div a^n = a^{m-n}$ $(a^b)^c = = a^{bc} = a^{cb}$ $a^{-n} = \frac{1}{a^n}$ $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$	**Use of** $a^0 = 1$ $2^0 = 1$ **Use of** $a^m \times a^n = a^{m+n}$ $5^1 \times 5^3 = 5^{1+3}$ Using $a^m \times a^n = a^{m+n}$ $= 5^4$ $= 5 \times 5 \times 5 \times 5$ $= 625$ **Use of** $a^m \div a^n = a^{m-n}$ $5(y^9 \div y^5) = 5(y^{9-5})$ Using $a^m \div a^n = a^{m-n}$ $= 5y^4$ **Use of** $(a^b)^c = = a^{bc} = a^{cb}$ $(y^2)^6 = y^{2\times 6}$ Using $(a^m)^n = a^{mn}$ $= y^{12}$ **Use of** $a^{-n} = \frac{1}{a^n}$ $2^{-2} = \frac{1}{2^2}$ *Using* $a^{-m} = \frac{1}{a^m}$ $= \frac{1}{4}$ **Use of** $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ $125^{2/3} = \left(\sqrt[3]{125}\right)^2$ Using $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ $= 5^2$ Recognize cube root of 125 is 5. $= 25$

<table>
<tr><th colspan="2">Algebra</th></tr>
<tr><th colspan="2">Product of two brackets</th></tr>
<tr><th>Points to Remember</th><th>Illustration/ Example</th></tr>
<tr><td>Find the product of two algebraic expressions using the distributive law
$(a + b)(c + d) = a(c + d) + b(c + d)$
$= ac + ad + bc + bd$

$(a + b)(c + d + e) = a(c + d + e) + b(c + d + e)$
$= ac + ad + ae + bc + bd + be$</td><td>By applying the distributive law
$(x + 4)(x + 3) = x(x + 3) + 4(x + 3)$
$= x^2 + 3x + 4x + 12$
$= x^2 + 7x + 12$</td></tr>
</table>

<table>
<tr><th colspan="2">Algebra</th></tr>
<tr><th colspan="2">Factorization of Simple expressions</th></tr>
<tr><th>Points to Remember</th><th>Illustration/ Example</th></tr>
<tr><td>In mathematics, factorization is the decomposition of an expression into a product of factors, which when multiplied together gives the original. There are four common ways to factorize an expression:

1) Removing a single common factor
e.g. $ab + ac - ad = a(b + c - d)$
$2a + 6b + 24c = 2(a + 3b + 12c)$

2) Quadratic factorization
$a^2 + 2ab + b^2 = (a + b)(a + b)$

3) Factorizing by grouping- Group the terms in pairs so that each pair of terms has a common factor
$am + cn + dn + bm$
$= am + bm + an + bn$
$= m(a+b) + n(a+b)$ (a+b) is now a common factor
$= (a+b)(m+n)$

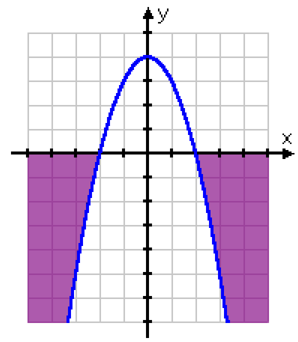
4) Difference of two squares
$(a+b)(a-b) = a^2 - b^2$</td><td>Removing a common factor:
$2y + 6 = 2(y + 3)$
$3y^2 + 12y = 3y(y + 4)$

Quadratic factorization
$x^2 + 4x + 3 = (x + 3)(x + 1)$

Factorizing by grouping
$xy - 4y + 3x - 12$
$= (xy - 4y) + (3x - 12) = y(x - 4) + 3(x - 4)$
$= (x - 4)(y + 3)$

$x^3 + 2x^2 + 8x + 16$
$= (x^3 + 2x^2) + (8x + 16) = x^2(x + 2) + 8(x + 2)$
$= (x + 2)(x^2 + 8)$

Difference of two squares
$4x^2 - 9 = (2x)^2 - (3)^2 = (2x+3)(2x - 3)$</td></tr>
</table>

<table>
<tr><td colspan="2">Algebra</td></tr>
<tr><td colspan="2">Solving quadratic inequalities</td></tr>
<tr><td>Points to Remember</td><td>Illustration/ Example</td></tr>
<tr><td>To solve a quadratic inequality:
1) Find the values of x when y = 0
2) In between these values of x , are intervals where the y values are either
greater than zero (>0), or
less than zero (<0)
3) To determine the interval either:
Draw the graph or
Pick a test value to find out which it is (>0 or <0)</td><td>Solve $-x^2 + 4 < 0$
Find out where the graph crosses the x-axis
$-x^2 + 4 = 0$
$x^2 – 4 = 0$
$(x + 2)(x – 2) = 0$
$x = -2$ or $x = 2$
To solve the original inequality, I need to find the intervals where the graph is below the axis i.e the y values are less than zero.


Then the solution is clearly: $x < -2$ or $x > 2$</td></tr>
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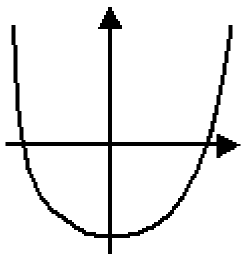
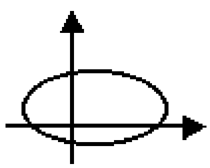
Relations, Functions and Graphs

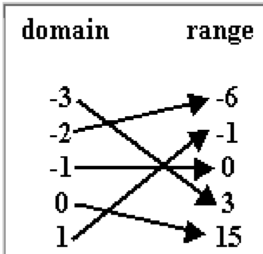
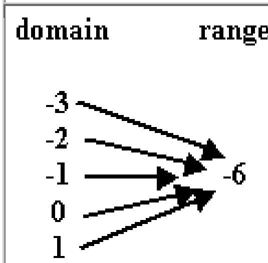
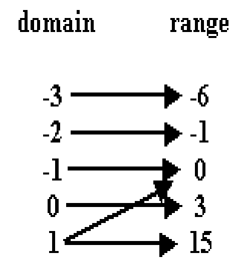
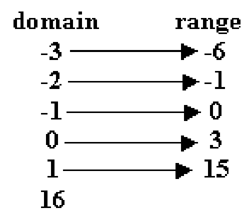
Relations and Functions

Points to Remember	**Illustration/ Example**

	This graph shows a function, because there is no vertical line that will cross this graph twice.
	This graph does not show a function, because any number of vertical lines will intersect this oval twice. For instance, the *y*-axis intersects (crosses) the line twice.

* A relation is a set of ordered pairs in which the first set of elements is called the domain and the second set of elements the range or co-domain
* Relations can be expressed in three ways: as expressions; as maps or diagrams; or as graphs
*A function is a mathematical operation that assigns to each input number or element, exactly one output number or value
* Maximum and minimum points on a graph are found where the slope of the curve is zero

Given the graph of a relation, if you can draw a vertical line that crosses the graph in more than one place, then the relation is not a function. Here are a couple examples:

	This is a function. There is only one *y* for each *x*; there is only one arrow coming from each *x*.
	This *is* a function! There is only one arrow coming from each *x*; there is only one *y* for each *x*
	This one is not a function: there are *two* arrows coming from the number 1; the number 1 is associated with two *different* range elements. So this is a relation, but it is not a function.
	Each element of the domain has a pair in the range. However, what about that 16? It *is* in the domain, but it has no range element that corresponds to it! This is neither a function nor a relation

<table>
<tr><td colspan="2">Relations, Functions and Graphs</td></tr>
<tr><td colspan="2">Composite Functions & Inverses</td></tr>
<tr><td>Points to Remember</td><td>Illustration/ Example</td></tr>
<tr><td>"Function Composition" is applying one function to the results of another:

The result of f( ) is sent through g( )

It is written: $(g \circ f)(x)$

Which means: $g(f(x))$</td>
<td>Given $f(x) = 2x + 3$ and $g(x) = -x^2 + 5$, find $(f \circ g)(x)$.
$(f \circ g)(x) = f(g(x))$
$= f(-x^2 + 5)$
$= 2(\qquad) + 3$ … setting up to insert the input formula
$= 2(-x^2 + 5) + 3$
$= -2x^2 + 10 + 3$
$= \mathbf{-2x^2 + 13}$

Given $f(x) = 2x + 3$ and $g(x) = -x^2 + 5$, find $(g \circ f)(x)$.
$(g \circ f)(x) = g(f(x))$
$= g(2x + 3)$
$= -(\qquad)^2 + 5$ … setting up to insert the input
$= -(2x + 3)^2 + 5$
$= -(4x^2 + 12x + 9) + 5$
$= -4x^2 - 12x - 9 + 5$
$= \mathbf{-4x^2 - 12x - 4}$

Find $(f \circ g)(2)$ using $(f \circ g)(x) = -2x^2 + 13$

$(f \circ g)(2) = -2(2)^2 + 13 = -8 + 13 = 5$
OR
Find $g(2) = -2^2 + 5 = -4 + 5 = 1$
Then $f[g(2)] = 2(1) + 3 = 5$</td></tr>
<tr><td>The inverse of a function has all the same points as the original function, except that the x's and y's have been reversed.

For instance, supposing your function is made up of these points: { (1, 0), (-3, 5), (0, 4) }. Then the inverse is given by this set of points:
{ (0, 1), (5, -3), (4, 0) }</td>
<td>Given $f(x) = 2x - 1$ and $g(x) = (\frac{1}{2})x + 4$,
find
i) $f^{-1}(x)$,
ii) $g^{-1}(x)$,
iii) $(f \circ g)^{-1}(x)$, and
iv) $(g^{-1} \circ f^{-1})(x)$.

First, find $f^{-1}(x)$, $g^{-1}(x)$, and $(f \circ g)^{-1}(x)$:

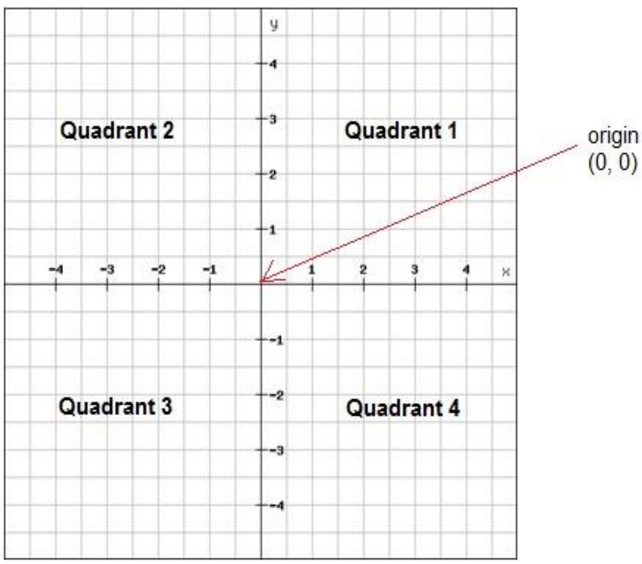
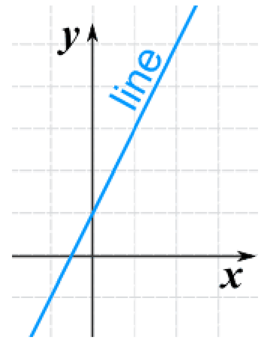
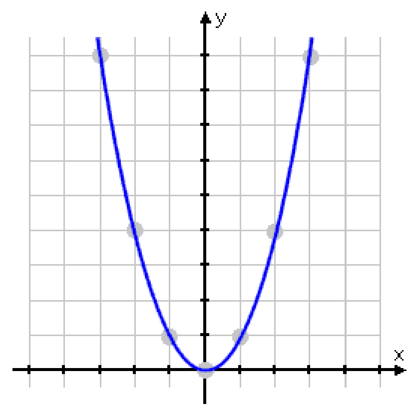
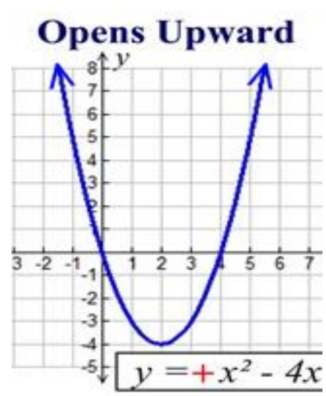
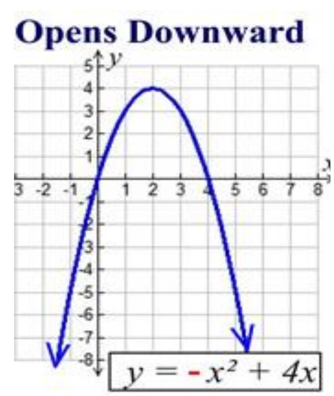
Inverting $f(x)$: $f(x) = 2x - 1$

Let $y = 2x - 1$
Interchange $x = 2y - 1$
Make y the subject
$x + 1 = 2y$
$\frac{x+1}{2} = y$
Hence,
$f^{-1}(x) = \frac{x+1}{2}$</td></tr>
</table>

Relations, Functions and Graphs	
Composite Functions & Inverses	
Points to Remember	**Illustration/ Example**
	Inverting g(x): $g(x) = \frac{1}{2}x + 4$ Let $y = \frac{1}{2}x + 4$ Interchange $x = \frac{1}{2}y + 4$ Make y the subject $x - 4 = \frac{1}{2}y$ $2(x - 4) = y$ $2x - 8 = y$ Hence $g^{-1}(x) = 2x - 8$ Finding the composite function: $(f \circ g)(x) = f[g(x)] = f[\frac{1}{2}x + 4]$ $= 2[\frac{1}{2}x + 4] - 1 = x + 8 - 1 = x + 7$ Inverting the composite function: $(f \circ g)(x) = x + 7$ *Let* $y = x + 7$ *Interchange* $x = y + 7$ Make y the subject $x - 7 = y$ **$(f \circ g)^{-1}(x) = x - 7$** Now compose the inverses of $f(x)$ and $g(x)$ to find the formula for $(g^{-1} \circ f^{-1})(x)$: $(g^{-1} \circ f^{-1})(x) = g^{-1}[f^{-1}(x)]$ $= g^{-1}\left(\frac{x+1}{2}\right)$ $= 2\left(\frac{x+1}{2}\right) - 8$ $= (x + 1) - 8$ Hence, **$(g^{-1} \circ f^{-1})(x) = x - 7$** The inverse of the composition $(f \circ g)^{-1}(x)$ gives the same result as does the composition of the inverses $(g^{-1} \circ f^{-1})(x)$. We therefore conclude that **$(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$**

Relations, Functions and Graphs

Introduction to Graphs

Points to Remember	**Illustration/ Example**
*In a graph, every point on the x-y plane can be labeled to indicate its position. * The coordinate axes divide the plane to four regions called quadrants * Graphs can be plotted from either linear equations or quadratic equations 	y = 2x + 1  $y = x^2$ 
The graphs of quadratic functions, $f(x) = ax^2 + bx + c$, are called parabolas. Parabolas may open upward or downward. If the sign of the leading coefficient, a, is positive ($a > 0$), the parabola opens upward. If the sign of the leading coefficient, a, is negative ($a < 0$), the parabola opens downward.	

Relations, Functions and Graphs	
Introduction to Graphs	
Points to Remember	**Illustration/ Example**

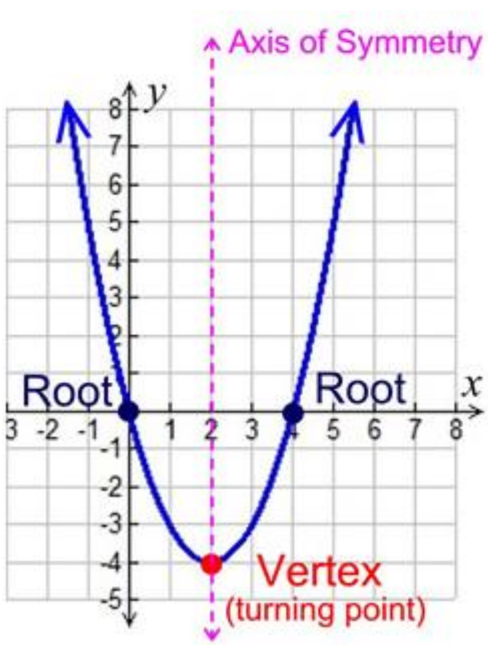
Parts of the quadratic graph:

The bottom (or top) of the U is called the vertex, or the turning point. The vertex of a parabola opening upward is also called the minimum point. The vertex of a parabola opening downward is also called the maximum point.

The x-intercepts are called the roots, or the zeros. To find the x-intercepts, set $ax^2 + bx + c = 0$.

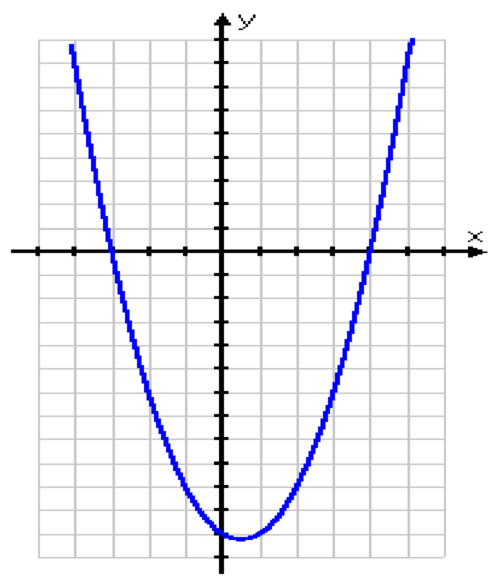
The ends of the graph continue to positive infinity (or negative infinity) unless the domain (the x's to be graphed) is otherwise specified.

The parabola is symmetric (a mirror image) about a vertical line drawn through its vertex (turning point).



Plot $y = x^2 - x - 12$ **for** $-4 \leq x \leq 5$

x	$y = x^2 - x - 12$
-4	$(-4)^2 - (-4) - 12 = 16 + 4 - 12 = 8$
-3	$(-3)^2 - (-3) - 12 = 9 + 3 - 12 = 0$
-2	$(-2)^2 - (-2) - 12 = 4 + 2 - 12 = -6$
-1	$(-1)^2 - (-1) - 12 = 1 + 1 = 12 = -10$
0	$(0)^2 - (0) - 12 = 0 - 0 - 12 = -12$
1	$(1)^2 - (1) - 12 = 1 - 1 - 12 = -12$
2	$(2)^2 - (2) - 12 = 4 - 2 - 12 = -10$
3	$(3)^2 - (3) - 12 = 9 - 3 - 12 = -6$
4	$(4)^2 - (4) - 12 = 16 - 4 - 12 = 0$
5	$(5)^2 - (5) - 12 = 25 - 5 - 12 = 8$



From the graph:

1) The values of x for which f(x) = 0 are x = -3 and x = 4 (roots or x intercept)
2) The value of x for which f(x) is minimum is x = $\frac{1}{2}$ (line of symmetry)
3) The minimum value of f(x) = -12.25
   The y-intercept is -12 (when x = 0; y= -12)

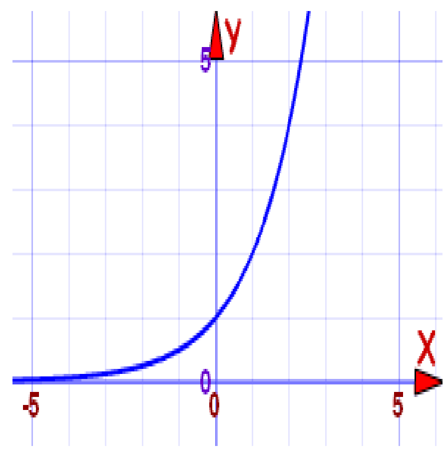
Relations, Functions and Graphs	
Introduction to Graphs	
Points to Remember	**Illustration/ Example**
Standard form A quadratic function is written as $y = ax^2 + bx + c$ **Roots** **Can be found by factorization.** It can also be found using the quadratic formula which gives the location on the x-axis of the two roots and will only work if ***a*** is non-zero. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **Axis of symmetry** $x = \frac{-b}{2a}$ **Completing the Square** When f(x) is written in the form $y= a(x - h)^2 + k$ (h, -k) is the maximum or minimum point **The y-intercept** is found by asking the question: When x = 0, what is y?	By Calculation: 1) The values of x for which f(x) = 0 a = 1, b = -1, c = -12 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-1 \pm \sqrt{(-1)^2 - 4(1)(-12)}}{2(1)}$ $x = \frac{-1 \pm \sqrt{1 + 48}}{2} = \frac{-1 \pm \sqrt{49}}{2} = \frac{-1 \pm 7}{2}$ $x = \frac{-1+7}{2} = \frac{6}{2} = 3$ or $x = \frac{-1-7}{2} = \frac{-8}{2} = -4$ 2) The value of x for which f(x) is minimum $x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$ 3) The minimum value of f(x) - complete the square or i.e. write f(x) in the form $y = a(x - h)^2 + k$ $y = x^2 - x - 12$ $y = x^2 - x + \left(\frac{1}{2}\right)^2 - 12 - \left(\frac{1}{2}\right)^2$ $y = \left(x - \frac{1}{2}\right)\left(x - \frac{1}{2}\right) - 12 - \frac{1}{4}$ $y = \left(x - \frac{1}{2}\right)^2 - \frac{49}{4}$ $a(x - h)^2 + k = \left(x - \frac{1}{2}\right)^2 - \frac{49}{4}$ This implies that $h = \frac{1}{2}$ and $k = \frac{-49}{4}$ So the minimum value of f(x) is $\frac{-49}{4}$ or -12.25 The minimum point is $\left(\frac{1}{2}, \frac{-49}{4}\right)$ 4) The y-intercept : When x = 0 $y = 0^2 - 0 - 12 = -12$

Relations, Functions and Graphs

Non-Linear Relations

Points to Remember	Illustration/ Example
Exponential functions involve exponents, where the variable is now the power. We encounter non-linear relations in the growth of population with time and the growth of invested money at compounded interest rates	Draw the graph of $y = 2^x$

x	$y = 2^x$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
4	$2^4 = 16$
5	$2^5 = 32$



Relations, Functions and Graphs	
Direct & Inverse Variation	
Points to Remember	**Illustration/ Example**
Direct Variation The statement " *y* varies directly as *x* ," means that when *x* increases, *y* increases *by the same factor*. y α x Introducing the constant of proportionality, k y = kx . **Other examples of direct variation:** The circumference of a circle is directly proportional to its radius.	**Example of Direct Variation:** If *y* varies directly as *x*, and *y* = 15 when *x* = 10, then what is *y* when *x* = 6? Find the constant of proportionality: y α x y = kx use (10, 15) 15 = k (10) $\frac{15}{10}$ = k $\frac{3}{2}$ = k Therefore the equation becomes y = $\frac{3}{2}$x Substitute x = 6 y = $\frac{3}{2}$ (6) y = 9 Solution (6, 9)
Inverse Variation Two quantities are inversely proportional if an increase in one quantity leads to a reduction in the other.	If *y* varies inversely as *x*, and *y* = 10 when *x* = 6 , then what is *y* when *x* = 15 ? y α $\frac{1}{x}$ y = $\frac{k}{x}$ 10 = $\frac{k}{6}$ k = 60 Therefore, the equation becomes y = $\frac{60}{x}$ When x = 15 y = $\frac{60}{15}$ = 4 Solution (6, 4)

Relations, Functions and Graphs

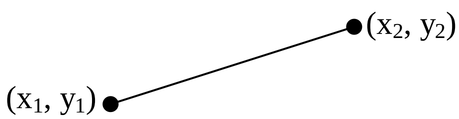
Coordinate Geometry

Points to Remember	Illustration/ Example

Points to Remember

a) Length of Line

One can use Pythagorean Theorem ($c^2 = a^2 + b^2$) to find the length of the third side (which is the hypotenuse of the right triangle):



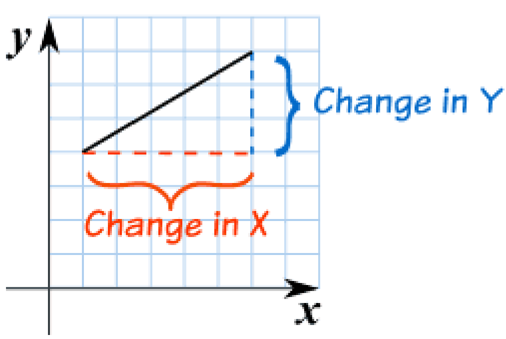
Length of line = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

b) Mid- point of Line

If you are given two numbers, you can find the number exactly between them by averaging them, by adding them together and dividing by two. If you need to find the point that is exactly halfway between two given points, just average the *x*-values and the *y*-values

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

c) Gradient of Line



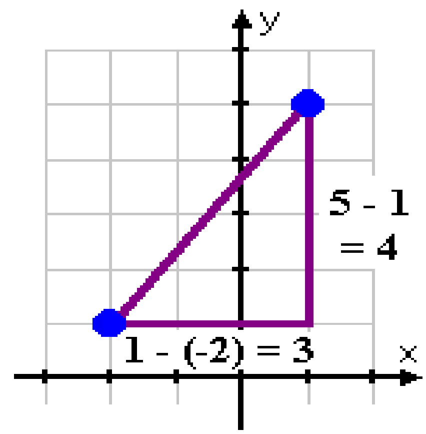
Gradient of the line passing through the points (x_1, y_1) and (x_2, y_2):

$$(m) = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{y_1 - y_2}{x_1 - x_2}$$

Illustration/ Example

A line joins the points (-2, 1) and (1, 5) find:

a) The length of the line
b) The midpoint of the line
c) The gradient of the line
d) The equation of the line
e) The gradient of any perpendicular to the line
f) The equation of the perpendicular bisector of the line



a) Length of line $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(1 - -2)^2 + (5 - 1)^2}$
$= \sqrt{(1 + 2)^2 + (4)^2}$
$= \sqrt{(3)^2 + (4)^2}$
$= \sqrt{9 + 16}$
$= \sqrt{25}$
= 5 units

b) Midpoint of line

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-2+1}{2}, \frac{5+1}{2}\right)$$
$$= \left(-\frac{1}{2}, \frac{6}{2}\right)$$
$$= \left(-\frac{1}{2}, 3\right)$$

Relations, Functions and Graphs

Coordinate Geometry

Points to Remember

d) Equation of line

The general **equation of a straight line** is y = mx + c, where m is the gradient, and y = c is the value where the **line** cuts the y-axis. This number c is called the intercept on the y-axis

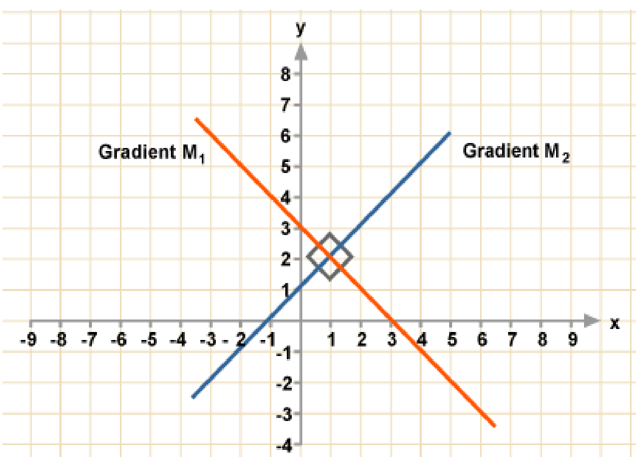
e) Gradients of Parallel Lines

On a graph, parallel lines have the same gradient. For example, y = 2x + 3 and y = 2x - 4 are parallel because they both have a gradient of 2.

f) Gradients of Perpendicular Lines

The product of the gradient of perpendicular lines will always be -1.

If lines are perpendicular, $M1 \times M2 = -1$



Illustration/ Example

c) Gradient of line

$$\frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{5-1}{1--2} = \frac{4}{3}$$

d) Equation of line:

y= mx + c

$y = \frac{4}{3}x + c$ (use the point (1, 5))

$5 = \frac{4}{3}(1) + c$

$5 - \frac{4}{3} = c$

$\frac{11}{3} = c$

Equation is $y = \frac{4}{3}x + \frac{11}{3}$

e) Gradient of any perpendicular to the line

$m_1 \times m_2 = -1$

$\frac{4}{3} \times m_2 = -1$

$m_2 = -1 \div \frac{4}{3}$

$m_2 = \frac{-3}{4}$

f) Equation of perpendicular bisector

y= mx + c

$y = \frac{-3}{4}x + c$ (use the midpoint $(-\frac{1}{2}, 3)$)

$3 = \frac{-3}{4}(-\frac{1}{2}) + c$

$3 = \frac{3}{8} + c$

$3 - \frac{3}{8} = c$

$\frac{21}{8} = c$

Equation of perpendicular bisector is

$y = \frac{-3}{4}x + \frac{21}{8}$

Relations, Functions and Graphs

Linear Programming

Points to Remember

Linear programming is the process of taking various linear inequalities relating to some situation, and finding the "best" value obtainable under those conditions. A typical example would be taking the limitations of materials and labor, and then determining the "best" production levels for maximal profits under those conditions.

Illustration/ Example

Suppose that the three (3) inequalities are related to some situation.

$$\begin{Bmatrix} x+2y\le 14 \\ 3x-y\ge 0 \\ x-y\le 2 \end{Bmatrix} \Rightarrow \begin{Bmatrix} y\le -\frac{1}{2}x+7 \\ y\le 3x \\ y\ge x-2 \end{Bmatrix}$$

These inequalities can be represented on a graph:

To draw the line y= $-\frac{1}{2}$x + 7:

When x = 0 , y = 7 and when y=0, x = 14

Therefore, coordinates on the line are (0, 7) and (14,0)

To draw the line y = 3x

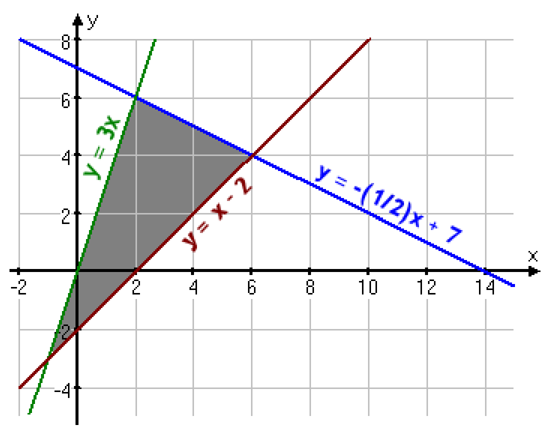
When x = 0, y = 0 and say when x = 2, y =6

Coordinates on the line are (0,0) and (2,6)

To draw the line y= x - 2

When x = 0, y = -2 and when y = 0, x = 2

Coordinates on the line are (0, -2) and (2,0)



Suppose the profit is given by the equation "P = 3*x* + 4*y*"

To find maximum profit:

The corner points are (2, 6), (6, 4), and (–1, –3).

For linear systems like this, the maximum and minimum values of the equation will always be on the corners of the shaded region. So, to find the solution simply plug these three points into "P = 3*x* + 4*y*".

(2, 6): P = 3(2) + 4(6) = 6 + 24 = 30

(6, 4): P = 3(6) + 4(4) = 18 + 16 = 34

(–1, –3): P = 3(–1) + 4(–3) = –3 – 12 = –15

Then **the maximum of *P* = 34 occurs at (6, 4)**,

and **the minimum of *P* = –15 occurs at (–1, –3)**.

Relations Functions and Graphs

Distance - Time Graphs

Points to Remember

<u>Speed, Distance and Time</u>

The following is a basic but important formula which applies when speed is constant (in other words the speed doesn't change)

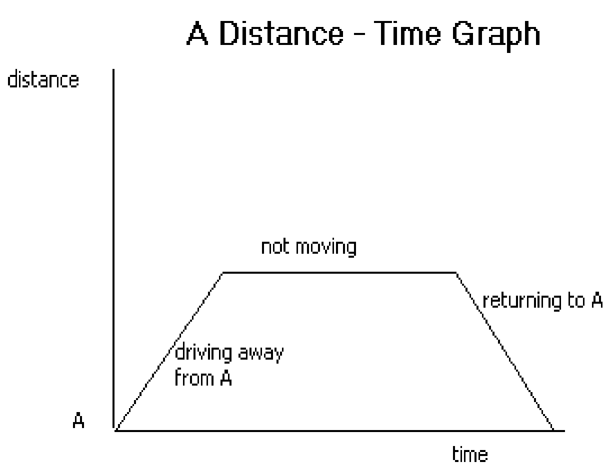
$$\text{Speed} = \frac{Distance}{Time}$$

If speed does change, the average (mean) speed can be calculated:

$$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time Taken}}$$

<u>Distance - Time Graphs</u>

These have the distance from a certain point on the vertical axis and the time on the horizontal axis. The velocity can be calculated by finding the gradient of the graph. If the graph is curved, this can be done by drawing a chord and finding its gradient (this will give average velocity) or by finding the gradient of a tangent to the graph (this will give the velocity at the instant where the tangent is drawn).



Illustration/ Example

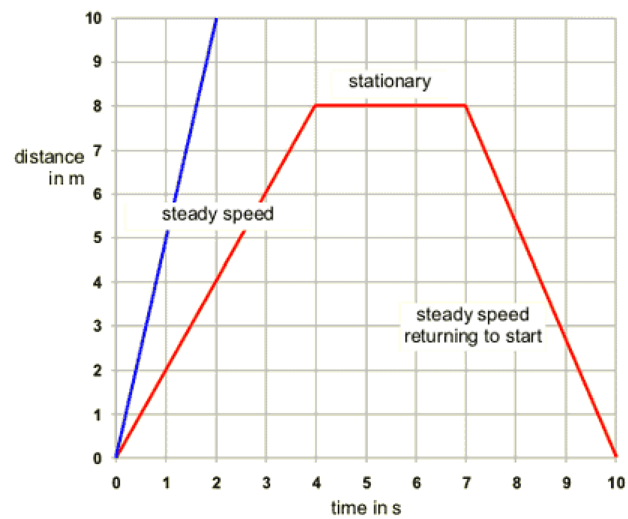
Example

a) Jane runs at an average speed of 12.5 m/s in a race journey of 500 metres. How long does she take to complete the race?

To find a time, we need to divide distance by speed.
500 metres ÷ 12.5 m/s = 40 secs

b) Chris cycles at an average speed of 8 km/h.
If he cycles for 6½ hours, how far does he travel?

To find a distance, we need to multiply speed by time.
8 km/h × 6.5 hours = 52 km



a) Change 15km/h into m/s.

$$15 \text{ km/h} = \frac{15\ km}{1\ hour}$$

$$= \frac{15\ km}{60\ min}$$

$$= \frac{15000\ m}{3600\ secs}$$

$$= 4\frac{1}{6} \text{ m/s}$$

Units

When using any formula, the units must all be consistent. For example speed could be measured in ***m/s***, in terms of distance in metres and time in seconds **or** in ***km/h*** in terms of distance in kilometres and time in hours.

In calculations, units must be consistent, so if the units in the question are not all the same e.g. ***m/s***, and ***km/h***, then you must first convert all to the same unit at the start of solving the problem.

b) Example If a car travels at a speed of 10m/s for 3 minutes, how far will it travel?
   i. Firstly, change the 3 minutes into 180 seconds, so that the units are consistent.
   ii. Now rearrange the first equation to get distance = speed × time.
   iii. Therefore distance travelled = 10m × 180 = 1800m = 1.8 km

c) A car starts from rest and within 10 seconds is travelling at 10m/s. What is its acceleration?

$$\text{Acceleration} = \frac{change\ in\ velocity}{time} = \frac{10}{10} = 1\ \text{m/s}^2$$

d) What is the speed represented by the steeper line?

$$\text{Speed} = \frac{10-0}{2-0} = \frac{10}{2} = 5\text{ms}^{-1}$$

Relations Functions and Graphs

Velocity - Time Graphs

Points to Remember	Illustration/ Example

<u>Velocity and Acceleration</u>

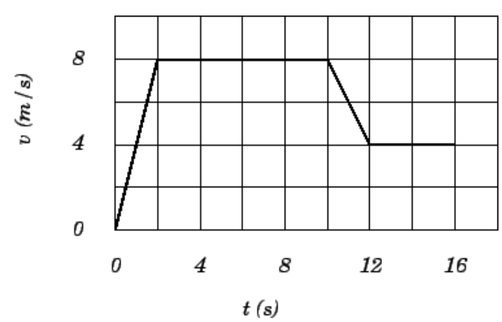
Velocity is the speed of a particle <u>and</u> its direction of motion (therefore velocity is a vector quantity, whereas speed is a scalar quantity).

When the velocity (speed) of a moving object is increasing we say that the object is *accelerating*. If the velocity decreases it is said to be decelerating. Acceleration is therefore the rate of change of velocity (change in velocity / time) and is measured in m/s².

Example

Consider the motion of the object whose velocity-time graph is given in the diagram.

a) What is the acceleration of the object between times $t = 0$ and $t = 2$?
b) What is the acceleration of the object between times $t = 10$ and $t = 12$?
c) What is the net displacement of the object between times $t = 0$ and $t = 16$?



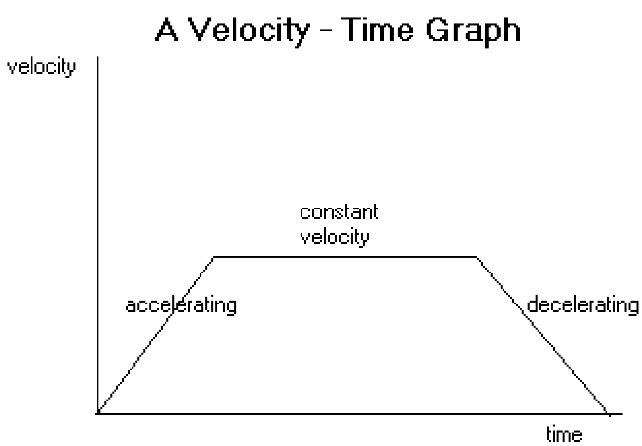
Relations Functions and Graphs

Velocity - Time Graphs

Points to Remember

Velocity-Time Graphs/ Speed-Time Graphs

A velocity-time graph has the velocity or speed of an object on the vertical axis and time on the horizontal axis.
The distance travelled can be calculated by finding the area under a velocity-time graph. If the graph is curved, there are a number of ways of estimating the area.
Acceleration is the gradient of a velocity-time graph and on curves can be calculated using chords or tangents.



The distance travelled is area under graph.
The acceleration and deceleration can be found by finding the gradient of the lines.

Illustration/ Example

a) The velocity-time graph is a straight-line between t=0 and t=2, indicating constant acceleration during this time period. Hence,

$$a = \frac{change\ in\ velocity}{change\ in\ time} = \frac{8-0}{2} = 4\ ms^{-2}$$

b) The velocity-time graph graph is a straight-line between t = 10 and t = 12 , indicating constant acceleration during this time period. Hence,

$$a = \frac{change\ in\ velocity}{change\ in\ time} = \frac{4-8}{2} = -2\ ms^{-2}$$

The negative sign indicates that the object is decelerating.

c) The net displacement between times t = 0 and t = 16 equals the area under the velocity-time curve, evaluated between these two times. Recalling that the area of a triangle is half its width times its height, the number of grid-squares under the velocity-time :

= Area of triangle + Area of Square + Area of Trapezium + Area of Square

$= \frac{1}{2}(b)(h) + (s \times s) + \frac{1}{2}(h)(a+b) + (s \times s)$

$= \frac{1}{2}(2)(8) + (8x8) + \frac{1}{2}(2)(4+8) + (4x4)$

= 8 + 64 + 12 + 16 = 100 m

Statistics

Displaying data: Pie Chart, Bar Graph, Histogram and Line Graph

Points to Remember	Illustration/ Example
*** A Pie Chart** uses "pie slices" to show relative sizes of data	The table shows the result of a survey of your friends, to find out which kind of movie they liked best.

Draw

1) A Pie Chart
2) A bar Graph

to represent the information

Favorite Type of Movie				
Comedy	**Action**	**Romance**	**Drama**	**SciFi**
4	5	6	1	4

Total number of friends = 20

% liking Comedy: 4/20 x 100 = 20

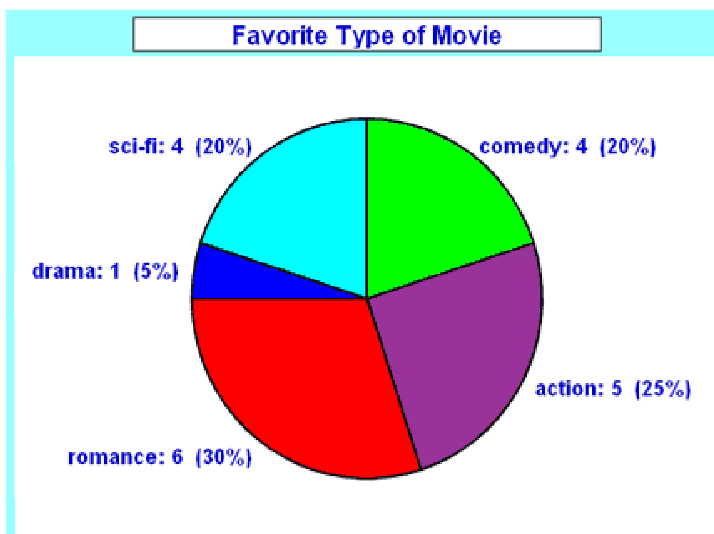
% liking Action: 5/20 x 100 = 25

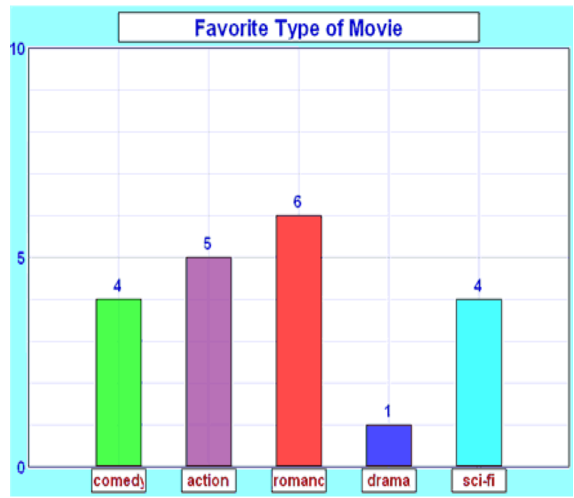
% liking Romance: 6/20 x 100 = 30

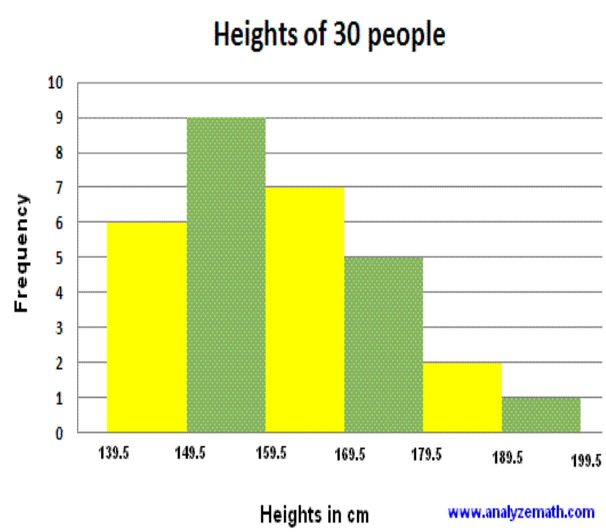
% liking Drama: 1/20 x 100 = 5

% liking SciFi: 4/20 x 100 = 20

Example: Pie Chart



<table>
<tr><td colspan="2">Statistics</td></tr>
<tr><td colspan="2">Displaying data: Pie Chart, Bar Graph, Histogram and Line Graph</td></tr>
<tr><td>Points to Remember</td><td>Illustration/ Example</td></tr>
<tr><td>* There are three basic types of Bar Charts: simple charts, proportionate charts and chronological charts</td><td>Example :Bar Graph

</td></tr>
<tr><td>.* Histograms are a great way to show results of continuous data, such as:
weight
height
how much time
etc.
The horizontal axis is continuous like a number line:

</td><td>The histogram below shows the heights (in cm) distribution of 30 people.
a) How many people have heights between 159.5 and 169.5 cm?
b) How many people have heights less than 159.5 cm?
c) How many people have heights more than 169.5cm?
d) What percentage of people have heights between 149.5 and 179.5 cm?


a) 7 people
b) 9 + 6 = 15 people
c) 5 + 2 + 1 = 8 people
d) (9 + 7 + 5)/ 30 x 100 = 70%</td></tr>
</table>

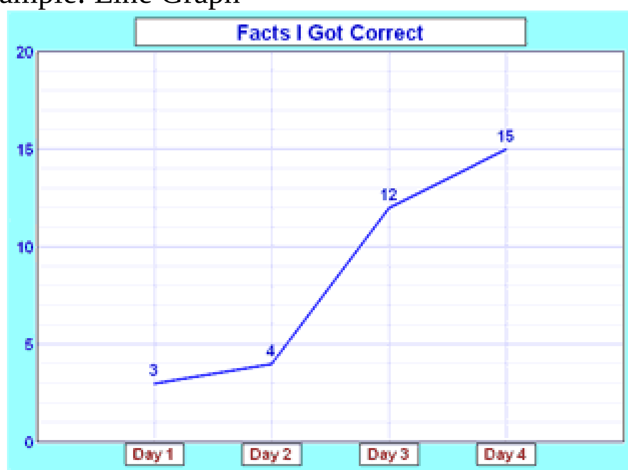
Statistics

Displaying data: Pie Chart, Bar Graph, Histogram and Line Graph

Points to Remember	Illustration/ Example
Line Graph - A graph that shows information that is connected in some way (such as change over time)	You are learning facts about dogs, and each day you do a short test to see how good you are. Draw a line graph to represent the information:

Facts I got Correct			
Day 1	**Day 2**	**Day 3**	**Day 4**
3	4	12	15

Example: Line Graph



Statistics

Frequency Distribution
Displaying data on the Bar Graph
Measure of Central Tendency – Mean, Median and Mode

Points to Remember	Illustration/ Example
The **frequency** of a particular data value is the number of times the data value occurs A **frequency table** is constructed by arranging collected data values in ascending order of magnitude with their corresponding frequencies The **Mean** is the **average** of the numbers. **Add up** all the numbers, then **divide by how many** numbers there are mean = $\frac{\Sigma xf}{\Sigma f}$	Rick did a survey of how many games each of 20 friends owned, and got this: 9, 15, 11, 12, 3, 5, 10, 20, 14, 6, 8, 8, 12, 12, 18, 15, 6, 9, 18, 11 a) Find the Mode b) Find the Median c) Show this data in a frequency table d) Calculate the mean e) Draw a histogram to represent the data

Statistics

Frequency Distribution

Displaying data on the Bar Graph

Measure of Central Tendency – Mean, Median and Mode

Points to Remember

To find the **Median**, place the numbers in value order and find the middle number (or the mean of the middle two numbers).

To find the **Mode**, or modal value, place the numbers in value order then count how many of each number. The Mode is the number which appears most often (there can be more than one mode):

Illustration/ Example

a) Mode is 12 (occurs most often)

b) To find the median, first order the data then find the mean of the 10^{th} and 11^{th} values:

3, 5, 6, 6, 8, 8, 9, 9, 10, **11, 11**, 12, 12, 12, 14, 15, 15, 18, 18, 20

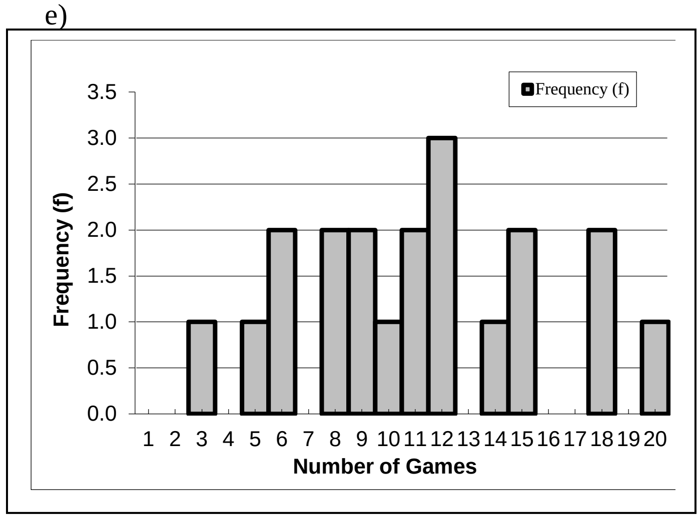
Median = $\frac{11+11}{2} = \frac{22}{2} = 11$

c) **Frequency table for the number of games owned.**

Number of games (x)	Tally	Frequency (f)	xf
3	|	1	3
5	|	1	5
6	||	2	12
8	||	2	16
9	||	2	18
10	|	1	10
11	||	2	22
12	|||	3	36
14	|	1	14
15	||	2	30
18	||	2	36
20	|	1	20
		Σf = 20	Σxf = 222

d) mean = $\frac{\Sigma xf}{\Sigma f} = \frac{222}{20} = 11.1$

e)



Statistics

Frequency Distribution
Displaying data on the Bar Graph
Measure of Central Tendency – Mean, Median and Mode

Points to Remember

* When the set of data values are spread out, it is difficult to set up a frequency table for every data value as there will be too many rows in the table. So we group the data into class intervals (or groups) to help us organize, interpret and analyze the data.

*The values are grouped in intervals (classes) that have the same width. Each class is assigned its corresponding frequency.

Class Limits
Each class is limited by an upper and lower limit

Class Width
The class width is the difference between the upper and lower limit of that particular class

Class Mark/ Mid-Interval Value
The class mark is the midpoint of each interval and is the value that represents the whole interval for the calculation of some statistical parameters and for the histogram

Estimating the Mode from a Histogram

1. Identify the tallest bar. This represents the modal class.
2. Join the tips of this bar to those of the neighbouring bars on either side, with the one on the left joined to that on the right and vice-versa. The lines used to join these tips cross each other at some point in this bar.
3. Drop a perpendicular line from the tip of the point where these lines meet to the base of the bar (horizontal axis). The point where it meets the base is the mode.

Illustration/ Example

The lengths of ribbon required to wrap 40 presents are as follows:

17	31	23	29	27	37	28	34	42	23
12	22	18	26	24	30	41	14	29	22
21	32	28	19	27	25	38	39	21	40
26	27	26	30	33	20	28	35	29	31

Construct a Grouped Frequency Table for the following data:

Length of Ribbon (cm)	Mid-Interval Value (x)	Frequency (f)	xf
6 – 10	8	0	0
11 – 15	13	2	26
16 – 20	18	4	72
21 – 25	23	8	184
26 – 30	28	14	392
31 – 35	33	6	198
36 – 40	38	4	152
41 – 45	43	2	86
		Σ f =40	**Σ xf = 1110**

We can estimate the **Mean** by using the **midpoints**

mean = $\frac{\Sigma xf}{\Sigma f} = \frac{1110}{40} = 27.75$

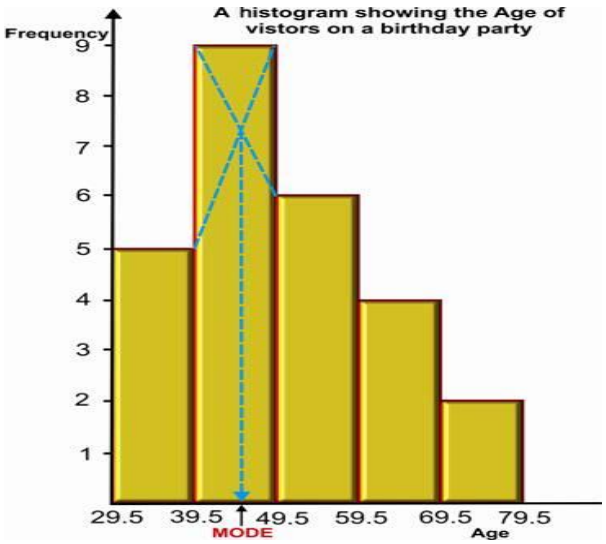
The **median** is the mean of the middle two numbers (the 20^{th} and 21^{th} values and they are both in the 26-30 group) ... The **median group** is 26-30. The median an also be found from a cumulative frequency curve (the second quartile value)

Statistics

Frequency Distribution
Displaying data on the Bar Graph
Measure of Central Tendency – Mean, Median and Mode

Points to Remember

4. Read off the value at the base using the estimation method.

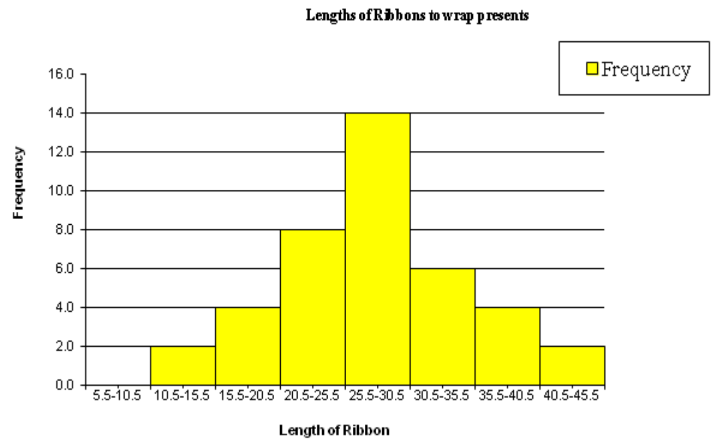


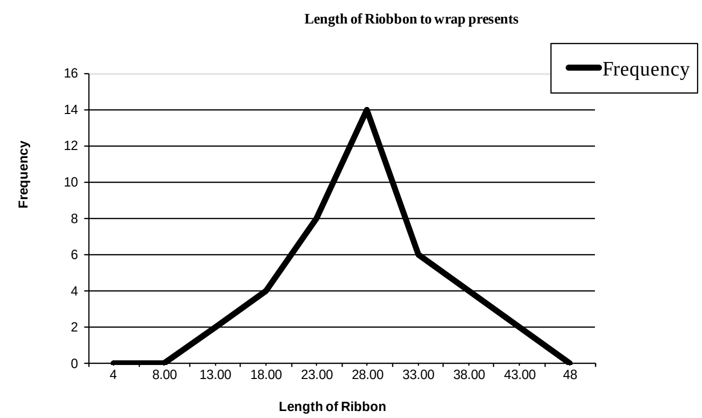
The mode is read off the horizontal axis.
In this case, the mode is 39.5 + 5.5 = 45.
The modal age of visitors is approximately 45 years.
To create a frequency polygon:

- Choose a *class interval.*
- Then draw an X-axis representing the values of the scores in your data.
- Mark the middle of each class interval with a tick mark, and label it with the middle value represented by the class.
- Draw the Y-axis to indicate the frequency of each class.
- Place a point in the middle of each class interval at the height corresponding to its frequency.
- Finally, connect the points.
- You should include one class interval below the lowest value in your data and one above the highest value.
- The graph will then touch the X-axis on both sides.

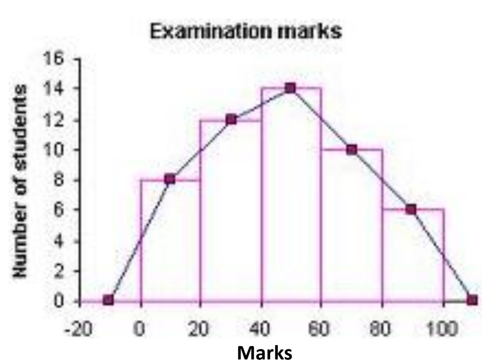
Illustration/ Example

The modal group (the group with the highest frequency), which is **26-30.** A single value for mode can be found from a **histogram.**





From the histogram, the mode is 28
Example: Frequency Polygon



Statistics

Cumulative Frequency Curve (Ogive)
Interquartile Range and Semi-Interquartile Range

Points to Remember

A **Cumulative Frequency Graph** is a graph plotted from a cumulative frequency table. A cumulative frequency graph is also called an **ogive** or **cumulative frequency curve**

The total of the frequencies up to a particular value is called the cumulative frequency

The **lower quartile** or first quartile (Q_1) is the value found at a quarter of the way through a set of data

The **median** or second quartile (Q_2) is the value found at half of the way through a set of data

The **upper quartile** (Q_3) is the value found at three quarters of the way through a set of data

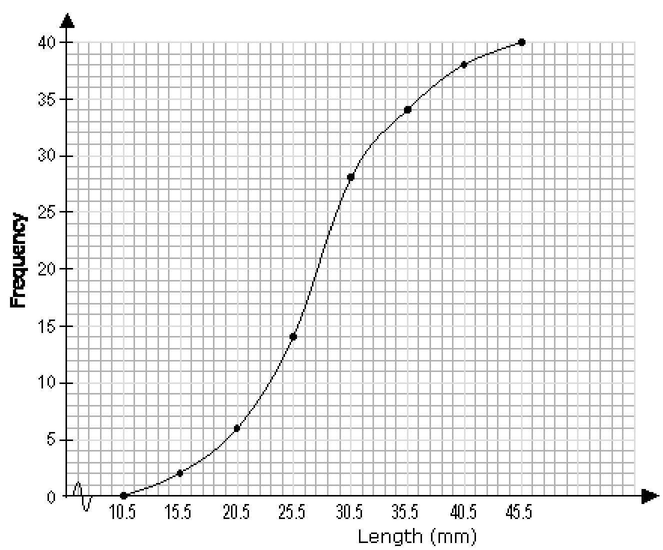
The **Interquartile range** is the difference between the upper and lower quartile: $Q_3 - Q_1$

Semi-interquartile range = ½ ($Q_3 - Q_1$)

Illustration/ Example

We need to add a class with 0 frequency before the first class and then find the upper boundary for each class interval

Length (*cm*)	**Frequency**	**Upper Class Boundary**	**Length (*x* cm)**	**Cumulative Frequency**
6 – 10	0	10.5	$x \le 10.5$	0
11 – 15	2	15.5	$x \le 15.5$	2
16 – 20	4	20.5	$x \le 20.5$	6
21 – 25	8	25.5	$x \le 25.5$	14
26 – 30	14	30.5	$x \le 30.5$	28
31 – 35	6	35.5	$x \le 35.5$	34
36 – 40	4	40.5	$x \le 40.5$	38
41 – 45	2	45.5	$x \le 45.5$	40
	Σ f = 40			



$Q_1 = 23.5$

$Q_2 = 27.5$

$Q_3 = 31.5$

Interquartile Range = Q3 – Q1 = 31.5- 23.5= 8

Semi-interquartile Range = ½ (Q3 – Q1) = ½ (8) = 4

Consumer Arithmetic

Ready Reckoner

Points to Remember

A ready reckoner is a table of numbers used to facilitate simple calculations, especially one for applying rates of discount, interest, charging, etc., to different sums

Illustration/ Example

The table shows an extract from a ready reckoner giving the price of N articles at 27 cents each.

N		N		N		N	
21	5.67	**63**	17.01	**105**	28.35	**500**	135.00
22	5.94	**64**	17.28	**106**	28.62	**525**	141.75
23	6.21	**65**	17.55	**107**	28.89	**550**	148.50
24	6.48	**66**	17.82	**108**	29.16	**600**	162.00
25	6.75	**67**	18.09	**109**	29.43	**625**	168.75
26	7.02	**68**	18.36	**110**	29.70	**650**	175.50
27	7.29	**69**	18.63	**111**	29.97	**700**	189.00
28	7.56	**70**	18.90	**112**	30.24	**750**	202.50
29	7.83	**71**	19.17	**113**	30.51	**800**	216.00
30	8.10	**72**	19.44	**114**	30.78	**900**	243.00

Use the table to find the cost of:

1) 23 articles at 27 cents each
2) 571 articles at 27 cents each
3) 61/4 m of material at 27 cents each
4) 72.9 kg of foodstuff at 27 cents per kilogram

1) Directly from the table, the cost is $6.21

2) From the table:

Cost of 500 articles = $135.00
Cost of 71 articles = $ 19.17
Cost of 571 articles = $154.17

3) $6\frac{1}{4}$ = 6.25. To use the tables we find the cost of 625 m to be:

$162.00 + $6.75 = $168.75.

Hence the cost of 6.25 m is $ $\frac{168.75}{100}$ = $ 1.69

4)The cost of 729 kg at 27 cents each is:

$189.00 + $7.83 = $196.83

Hence the cost of 72.9kg is $ $\frac{196.83}{10}$ = $ 19.68

Consumer Arithmetic	
Foreign Exchange Rates	
Points to Remember	**Illustration/ Example**
Foreign exchange, is the conversion of one country's currency into that of another	Amelia is going on a holiday to Italy, so she will have to purchase some euros (€). How many euros will she get for £375 if the exchange rate is £1 = €1.2769? Give your answer to the nearest euro. £1 = €1.2769 £375 = $\frac{1.2769}{1}$ x 375 = $ 478.8375 Change US$80 to TT$, given that TT$1.00 = US$6.35 US $6.35 = TT$ 1.00 US $1.00 = TT$ $\frac{1.00}{6.35}$ US $ 6.35 = TT$ $\frac{1.00}{6.35}$ x 80 = TT$ 12.59

Consumer Arithmetic	
Hire Purchase	
Points to Remember	**Illustration/ Example**
Goods purchased on hire purchase are paid for at regular intervals over a specified period of time. Sometimes the purchaser may pay a deposit, then the remainder (cash price- deposit + interest) is repaid at a number of regular intervals.	A bicycle can be bought for $160.00 cash or it can be bought on hire purchase by depositing 25% of the cash price, then paying the balance + 10% interest per annum (p.a.) on the balance in 12 monthly instalments. If the bicycle was sold on hire purchase determine the monthly repayments. Cash Price = $160.00 Deposit = $\frac{25}{100}$ x 160 = $40.00 Balance = $160.00 - $40.00 = $120.00 Interest on Balance = $\frac{10}{100}$ x 120 = $12.00 Total amount still to be paid = $120.00 + $12.00 = $132.00 Monthly repayment = $\frac{132}{12}$ = $ 11.00

Consumer Arithmetic	
Profit, Loss, Discount	
Points to Remember	**Illustration/ Example**
If an article is sold for more than it cost, then it is said to have been sold at a **profit** **Profit** = Selling Price – Cost Price **Profit %** $= \frac{\text{Profit}}{\text{Cost Price}} \times 100$ $= \frac{\text{Selling Price} - \text{Cost Price}}{\text{Cost Price}} \times 100$	1) A merchant bought a shirt for \$10.00 and sold it for \$13.00. a) Calculate the Profit b) Determine the percentage profit Profit = Selling price – Cost price = \$13.00 - \$10.00 = \$3.00 Profit % $= \frac{\text{Selling Price} - \text{Cost Price}}{\text{Cost Price}} \times 100$ $= \frac{\text{Profit}}{\text{Cost Price}} \times 100 = \frac{3}{10} \times 100 = 30\%$
If an article is sold for less than it cost, then it is said to have been sold at a **loss** **Loss %** $= \frac{\text{Cost Price} - \text{Selling Price}}{\text{Cost Price}} \times 100$ Profit is often expressed as a percentage of the cost price. This is called the **percentage profit**	2) A vase costing \$60.00 is sold for \$50.00. Find the percentage loss Loss = Cost price - Selling price = \$60.00 - \$50.00 = \$10.00 Loss % $= \frac{\text{Cost Price} - \text{Selling Price}}{\text{Cost Price}} \times 100$ $= \frac{\text{Loss}}{\text{Cost Price}} \times 100 = \frac{10}{60} \times 100 = 16\frac{2}{3}\%$
Percentage discount $= \frac{\text{Marked Price} - \text{Selling Price}}{\text{Marked Price}} \times 100$	3) A watch priced at \$160.00 is sold for \$140.00. a) Calculate the discount b) Determine the percentage discount Discount = Marked Price – Selling Price = \$160.00 - \$140.00 = \$20.00 Percentage discount $= \frac{\text{Marked Price} - \text{Selling Price}}{\text{Marked Price}} \times 100$ $= \frac{20}{160} \times 100 = 12\frac{1}{2}\%$ 4) A house was bought for \$60 000 and is sold for \$75000. What is the percentage profit? **Profit %** $= \frac{\text{Profit}}{\text{Cost Price}} \times 100$ $= \frac{\text{Selling Price} - \text{Cost Price}}{\text{Cost Price}} \times 100$ $= \frac{75000 - 60000}{60000} \times 100 = \frac{15000}{60000} \times 100 = 25\%$

Consumer Arithmetic	
Simple Interest	
Points to Remember	**Illustration/ Example**
Money deposited in a bank will earn interest at the end of the year. The sum of money deposited is called the principal. The interest is a percentage of the principal given by the bank for depositing with it. This percentage is called rate. If interest is always calculated on the original principal, it is called simple interest. Remember to change time given in months to years, by dividing by 12. Simple interest = $\frac{\mathbf{P \times R \times T}}{\mathbf{100}}$	Determine the simple interest on \$460 at 5% per annum for 3 years. Simple interest = $\frac{\mathbf{P \times R \times T}}{\mathbf{100}} = \frac{\mathbf{460 \times 5 \times 3}}{\mathbf{100}}$ = \$69.00 Simon wanted to borrow some money to expand his fruit shop. He was told he could borrow a sum of money for 30 months at 12% simple interest per year and pay \$1440 in interest charges. How much money can he borrow? T = $\frac{30}{12}$ = 2.5 years P = $\frac{\mathbf{SI \times 100}}{\mathbf{R \times T}} = \frac{\mathbf{1440 \times 100}}{\mathbf{12 \times 2.5}}$ = \$ 4800 Determine the time in which \$82 at 5% per annum will produce a simple interest of \$8.20 Time = $\frac{\mathbf{SI \times 100}}{\mathbf{P \times T}} = \frac{\mathbf{8.20 \times 100}}{\mathbf{82 \times 5}}$ = 2 years

Consumer Arithmetic	
Compound Interest	
Points to Remember	**Illustration/ Example**
A sum of money is invested at **compound interest**, when the interest at the end of the year (or period) is added to the principal, hence increasing the principal and increasing the interest the following year (or period). The principal plus the interest is called the amount. For compound interest, the interest after each year is added to the principal and the following year's interest is found from that new principal **Compound Interest**: $A = P(1 + r/100)^n$	Calculate the compound interest on \$640 at 5% per annum for 3 years. What is the Amount after three years?

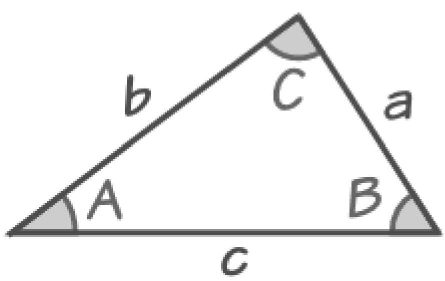
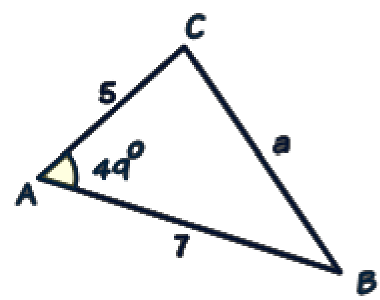
1^{st} Principal	640.00
1^{st} interest ($\frac{640 \times 5}{100}$ = \$32)	32.00
2^{nd} Principal	672.00
2^{nd} interest ($\frac{672 \times 5}{100}$ = \$33.60)	33.60
3^{rd} Principal	705.60
3^{rd} Interest $\frac{705.60 \times 5}{100}$ = \$35.28)	35.28
Amount	\$806.48

The compound interest for 3 years
= \$32.00 + \$ 33.60 + \$35.28 = \$100.88
Amount after 3 years is \$806.48

Consumer Arithmetic	
Mortgages	
Points to Remember	**Illustration/ Example**
A mortgage is a loan to finance the purchase of real estate, usually with specified payment periods and interest rates. The borrower (mortgagor) gives the lender (mortgagee) a right of ownership on the property as collateral for the loan.	1) Tim bought a house for 250,000. He makes a down payment of 15% of the purchase price and takes a 30-year mortgage for the balance. a) What is Tim's down payment? b) What is Tim's mortgage? **Downpayment** = Percent Down × Purchase Price $= \frac{15}{100}$ x \$250,000 = \$37500 **Amount of Mortgage** = Purchase Price − Down Payment = 250,000 − 37500 = 212500 2) If your monthly payment is 1200 dollars, what is the total interest charged over the life of the loan? **Total Monthly Payment** = Monthly payment × 12 × Number of years = \$1200 × 12 × 30 = \$432000 **Total Interest Paid** = Total Monthly Payment − Amount of Mortgage = \$432000 − \$212500 = \$219500

Consumer Arithmetic	
Rates and Taxes	
Points to Remember	**Illustration/ Example**
Taxes are 'calculated' sums of money paid to a government by to meet national expenditures • e.g. schools, hospitals, salaries, road networks **Gross Salary** is the figure before making other deductions. • **Tax-free allowance** - Working people do not pay tax on all their income. Part of their earnings is not taxed. A tax-free allowance is made for each dependent. Examples of dependents are : a wife, a young child, old father. • **Taxable income** is obtained after the tax-free allowance is subtracted from the gross salary • **Net** salary is the take home salary of the employee after paying taxes	Mr. Salandy's salary is \$22 000 per year. He has a personal allowance of \$2000, a marriage allowance of \$1000, a child allowance of \$800, national insurance of \$400 and an insurance allowance of \$300. A flat rate of 18% is paid on income tax. Determine his net salary. Personal allowance = 2000 Marriage allowance = 1000 Child allowance = 800 National insurance = 400 Insurance allowance = <u>300</u> Total Allowance = <u>4500</u> Taxable income = \$22000 - \$4500 = \$17 500 Income Tax = 18% of \$17500 = $\frac{18}{100}$ x 17,500 = \$3150. Net Salary = \$ 22 000 - \$3150 = \$18 850

Consumer Arithmetic	
Wages	
Points to Remember	**Illustration/ Example**
*Basic Week- Number of hours worked per week *Basic Rate- Amount of money paid per hour *Workers are paid wages and salaries. Wages can be paid fortnightly, weekly or daily. *Overtime- The money earned for extra hours beyond the basic week	1) A man works a basic week of 38 hours and his basic rate is $13.75 per hour. Calculate his total wage for the week Total wage for week= Basic Rate x Time = 13.75 x 38 = $522.50 2)John Williams works a 42 hour week for which he is paid a basic wage of $113.40. He works 6 hours overtime at time and a half and 4 hours at double time. Calculate his gross wage for the week. Basic hourly rate = $\frac{\$113.40}{42}$ = $2.70 Overtime rate at time and a half = 1 ½ x $2.70 = 4.05 For 6 hours at time and a half, Mr. William will earn $4.05 x 6 = $24.30 Overtime rate at double time = 2 x $2.70 = $5.40 For 4 hours at double time, Mr. William will earn $5.40 x 4 = $21.60 Gross Wage = $113.40 + 24.30 + 21.60 = $159.30

Trigonometry	
Cosine Rule	
Points to Remember	**Illustration/ Example**
When a triangle does not have a right angle, we can find the missing sides or angles using either the sine rule or the cosine rule It is used when two sides and an angle between them are given or all three sides are given **The Cosine Rule** is very useful for solving triangles: $$c^2 = a^2 + b^2 - 2ab\cos(C)$$  **a**, **b** and **c** are sides **C** is the angle opposite side c	This following examples will cover how to: • Use the Cosine Rule to find unknown sides and angles • Use the Sine Rule to find unknown sides and angles • Combine trigonometry skills to solve problems  $a^2 = b^2 + c^2 - 2bc \cos A$ $a^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos(49°)$ $a^2 = 25 + 49 - 70 \times \cos(49°)$ $a^2 = 74 - 70 \times 0.6560...$ $a^2 = 74 - 45.924... = 28.075$ $a = \sqrt{28.075...}$ $a = 5.298...$ a = **5.30** to 2 decimal places

Trigonometry	
Sine Rule	
Points to Remember	**Illustration/ Example**
The Sine Rule is also very useful for solving triangles: $$\frac{b}{\sin B} = \frac{a}{\sin A}$$ When two angles and any side are given or when two sides and an angle not between them are given	$\frac{b}{\sin B} = \frac{a}{\sin A}$ $\frac{5}{\sin B} = \frac{5.298}{\sin 49}$ $\sin B = (\sin(49°) \times 5) / 5.298...$ $\sin B = 0.7122...$ $B = \sin^{-1}(0.7122...)$ B = **45.4°** to one decimal place $C = 180° - 49° - 45.4°$ C = **85.6°** to one decimal place

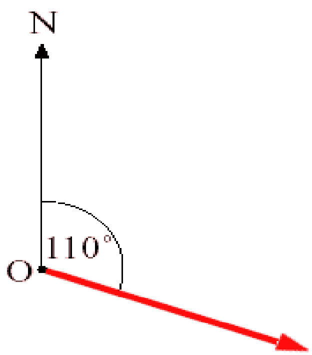
Trigonometry

Bearings

Points to Remember

Bearings are a measure of direction, with North taken as a reference.
If you are travelling North, your bearing is 000°, and this is usually represented as straight up on the page.
If you are travelling in any other direction, your *bearing* is measure clockwise from North.

Example
Look at the diagram below:

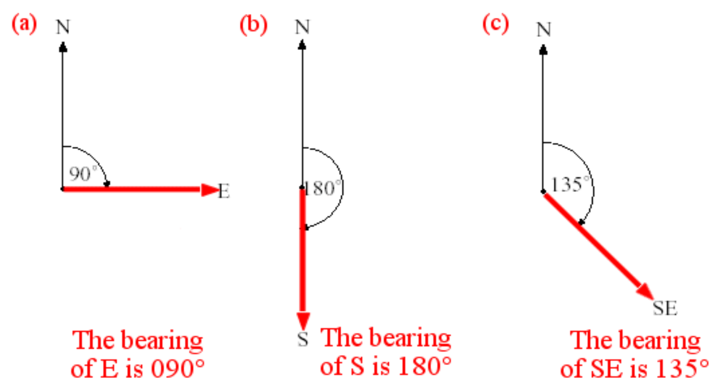


If you walk from O in the direction shown by the red arrow, you are walking on a bearing of 110 °.

Use simple trigonometrical ratios as well as the sine and cosine rules to solve problems involving bearings
Presents several problems and ask students decide whether to use the sine or cosine rule, or the trigonometric ratios

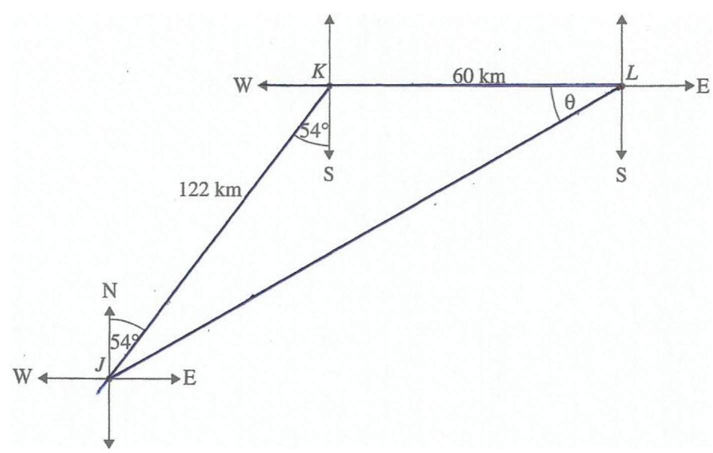
Illustration/ Example

1) Find the bearings for:
   (a) East (E)
   (b) South (S)
   (c) South-East (SE)



2) J, K and L are three sea ports. A ship began its journey at J, sailed to K, then to L and returned to J.
   The bearing of K from J is 054° and L is due east of K.
   JK= I22 km and KL= 60km.

   (i) Draw a clearly labelled diagram to represent the above information. Show on the diagram
   a) the north/south direction
   b) the bearing 054°
   c) the distances l22km and 60 km.



	(ii) Calculate a) the measure of angle JKL b) the distance JL c) the bearing of J from L (a) Required to calculate angle JKL, angle JKL = $90^o + 54^o$ $= 144^o$ (b) $JL^2 = JK^2 + KL^2 - 2\,(JK)(KL)\cos 144^o$ $= (122)^2 + (60)^2 - 2(122)(60)\cos 144^o$ $= 30328.008$ JL $= \sqrt{30328.008}$ JL $= 174.15$ km (c) The bearing of J from L $\frac{122}{\sin\theta} = \frac{174.149}{\sin 144^o}$ $\sin\theta = \frac{122 \times \sin 144^o}{174.149}$ $\theta = \sin^{-1}(0.4417)$ $\theta = 24.31^o$ $L = 270^o - 24.31^o$ $= 245.7^o$ to the nearest 0.1^o

Sets

Definitions and Notation

Points to Remember

*A set is a collection of identifiable elements or members that are connected in some way
* There are two types of sets: finite and infinite
*The symbol ∈ is used to show that an item is an element or member of a set
* A subset is represented by the symbol: ⊂ and is used to present part of a set separately.
* A universal set is made up of all elements from which all subsets will be pulled and is represented by the symbol ε or U
* Venn Diagrams are used to represent sets and the relationship between sets (Describe each region).
* Complements of a set B are represented by B′ and show members of a set that are NOT part of B.
* The intersection of two or more sets consists of those elements that are common to those sets
*The union of two or more sets consists of those elements that make up those sets.

Set Notation	Description	Meaning
A U *B*	"*A* union *B*"	everything that is in either of the sets
A ∩ B	"*A* intersect *B*"	only the things that are in both of the sets
A'	"*A* complement" or "not *A*"	everything in the universal set outside of *A*
B′	"*B* complement"	everything in *A* except for anything in its overlap with *B*
(*A* ∪ *B*)'	"not (*A* union *B*)"	everything outside *A* and *B*
(A ∩ B)'	"not (*A* intersect *B*)"	everything outside of the overlap of *A* and *B*

Illustration/ Example

Examples of sets are: a collection of coins; a pack of cards; all vowels in the English alphabet etc.

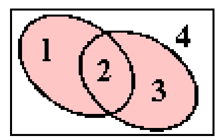
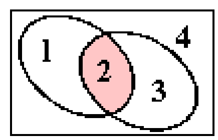
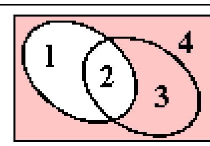
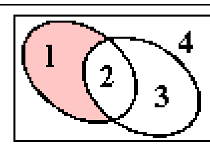
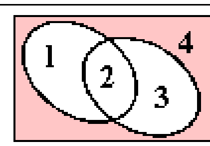
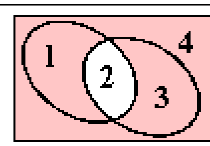
Examples of finite sets:
A= {All odd numbers between 1 and 10} = {3, 5, 7, 9}
V= {the vowels in the alphabet} = {a, e, i, o, u}

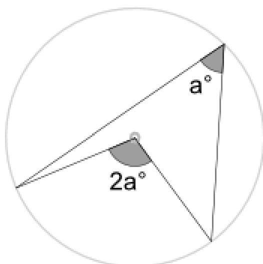
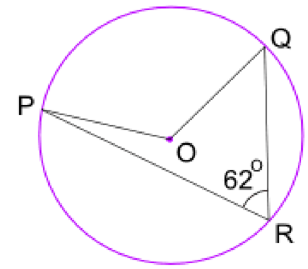
Examples of infinite sets:
A= {All natural numbers} **=** {1, 2, 3 …}
B= {All whole numbers} = {0, 1, 2, 3, 4 …}

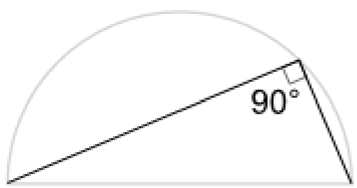
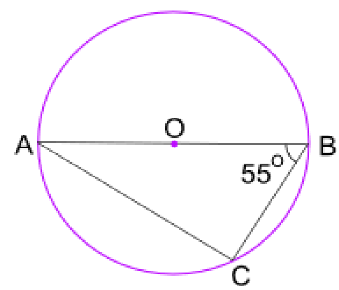
Use of symbol ∈ :
{2} ∈ {1, 2, 3, 4}

Use of symbol ⊂ :
{2, 3} ∈ {1, 2, 3, 4}

Set Notation:

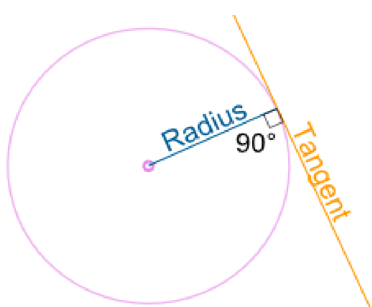
Set notation	Venn diagram	Set
A U *B*		{1, 2, 3}
A ∩ B		{2}
A'		{3, 4}
B′		{1}
(*A* U *B*)'		{4}
(A ∩ B)'		

<table>
<tr><th colspan="2">Circle Geometry</th></tr>
<tr><th colspan="2">Circle Theorems</th></tr>
<tr><th>Points to Remember</th><th>Illustration/ Example</th></tr>
<tr><td>The angle which an arc of a circle subtends at the centre of a circle is twice the angle it subtends at any point on the remaining part of the circumference.

</td><td>What is the size of Angle POQ? (O is circle's center)


Angle POQ = 2 × Angle PRQ = 2 × 62° = 124°</td></tr>
<tr><td>The angle in a semicircle is a right angle.

</td><td>What is the size of Angle BAC?


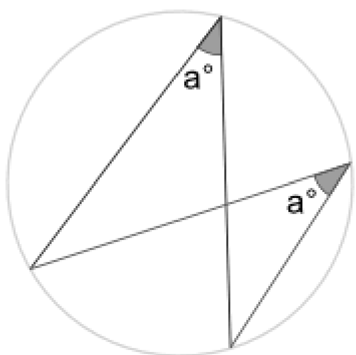
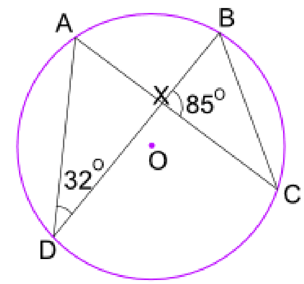
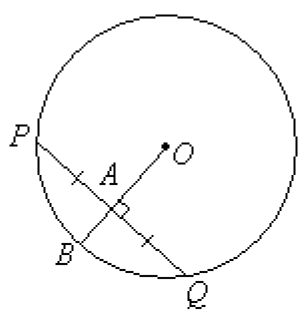
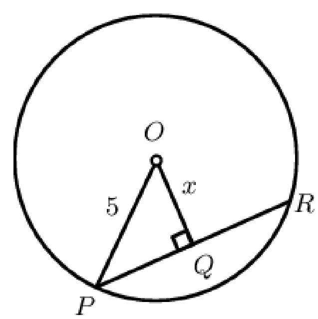
The Angle in the Semicircle Theorem tells us that Angle ACB = 90°

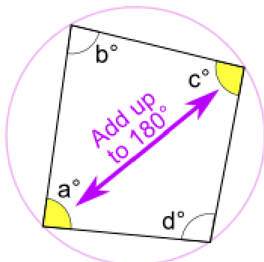
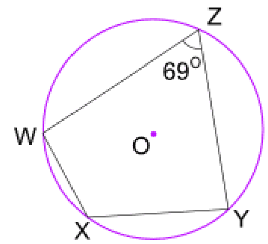
Now use the sum of angles of a triangle equals 180° to find Angle BAC:

Angle BAC + 55° + 90° = 180°
Angle BAC = 35°</td></tr>
<tr><td>A tangent of a circle is perpendicular to the radius of that circle at the point of contact.

</td><td></td></tr>
</table>

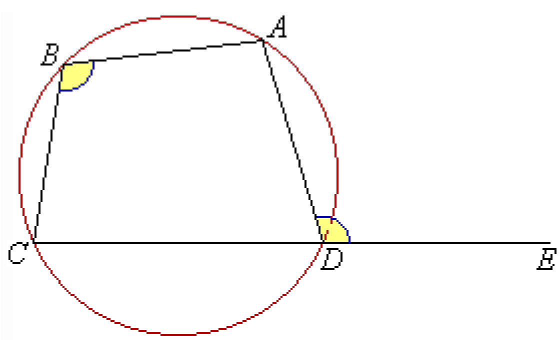
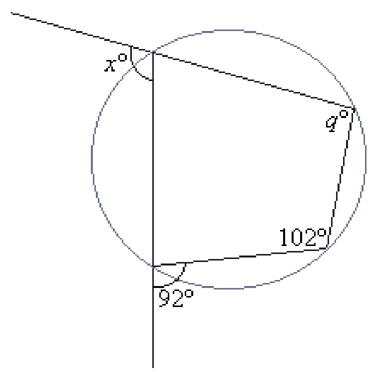
Circle Geometry

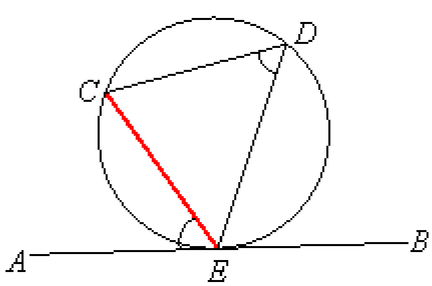
Circle Theorems

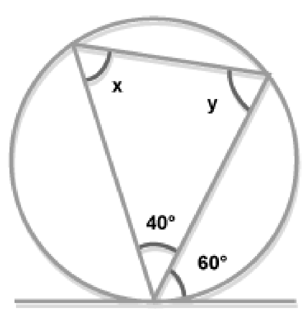
Points to Remember	**Illustration/ Example**
Angles in the same segment of a circle and subtended by the same arc are equal. 	What is the size of Angle CBX?  Angle ADB = 32° = Angle ACB. Angle ACB = Angle XCB. So in triangle BXC we know Angle BXC = 85°, and Angle XCB = 32° Now use sum of angles of a triangle equals 180° : Angle CBX + Angle BXC + Angle XCB = 180° Angle CBX + 85° + 32° = 180° Angle CBX = 63°
The line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord. 	Given that OQ is perpendicular to PR and $PR = 8$ units, determine the value of x  PQ = QR= 4 (perpendicular from centre bisects chord) In $\triangle OQP$: $PQ = 4$ $\quad$ ($\perp$ from centre bisects chord) $OP^2 = OQ^2 + QP^2$ $\quad$ (Pythagoras) $5^2 = x^2 + 4^2$ $\therefore x^2 = 25 - 16$ $x^2 = 9$ $x = 3$

<table>
<tr><td colspan="2">Circle Geometry</td></tr>
<tr><td colspan="2">Circle Theorems</td></tr>
<tr><td>Points to Remember</td><td>Illustration/ Example</td></tr>
<tr><td>The opposite angles of a cyclic quadrilateral are supplementary
• a + c = 180°
• b + d = 180°

</td><td>What is the size of Angle WXY?


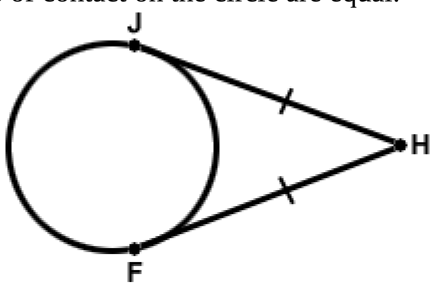
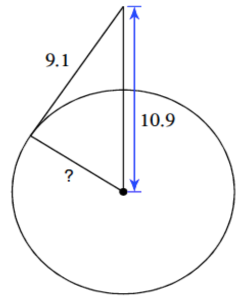
Opposite angles of a cyclic quadrilateral add to 180°

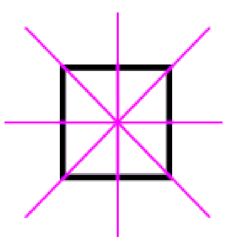
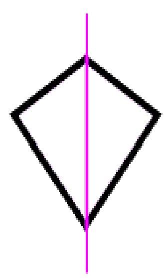
Angle WZY + Angle WXY = 180°
69° + Angle WXY = 180°
Angle WXY = 111°</td></tr>
<tr><td>The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle i.e. $\angle ADE = \angle ABC$

</td><td>

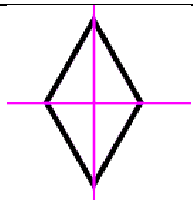
Find the values of x and q in the following diagram.
x = 102^o (exterior angle of a cyclic quadrilateral)
q = 92^o (exterior angle of a cyclic quadrilateral)</td></tr>
<tr><td>The angle between a tangent to a circle and a chord through the point of contact is equal to the angle in the alternate segment.


Angle CEA and angle CDE are equal.</td><td>Find the value of angles x and y


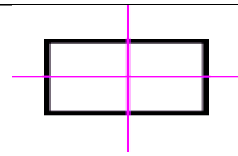
x = 60°
y = 80°
(using that the angles in a triangle add up to 180°)</td></tr>
</table>

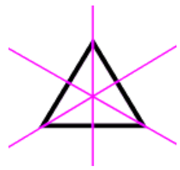
Circle Geometry	
Circle Theorems	
Points to Remember	**Illustration/ Example**
The lengths of two tangents from an external point to the points of contact on the circle are equal. 	Calculate the unknown length.  This is a right angled triangle because a tangent of a circle is perpendicular to the radius of that circle at the point of contact. Therefore, use Pythagoras' theorem $?^2 = 10.9^2 - 9.1^2 = 118.81 - 82.81 = 36$ $? = 6.0$

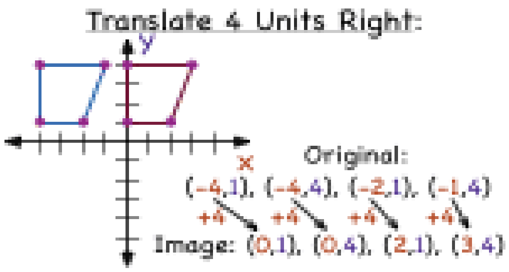
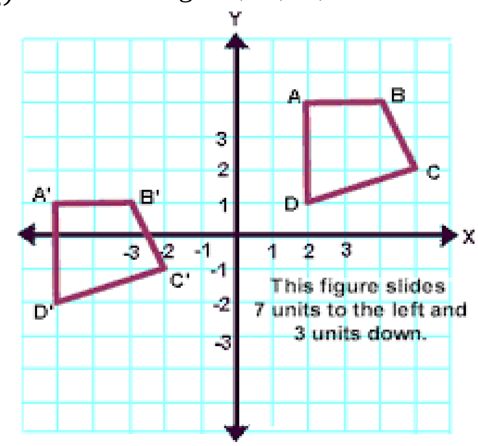
Symmetry	
Lines of Symmetry	
Points to Remember	**Illustration/ Example**
Definition: • A line of symmetry is an imaginary line that can divide an object in equal opposite parts. The line of symmetry is also called the 'mirror line'; it can be horizontal, vertical or at any angle. • Some shapes have no lines of symmetry; • A circle has an infinite number of lines of symmetry.  A square has 4 lines of symmetry	Identify and determine the number of lines of symmetry in the following shapes: a) Kite b) Rectangle c) Triangle  A kite has 1 line of symmetry

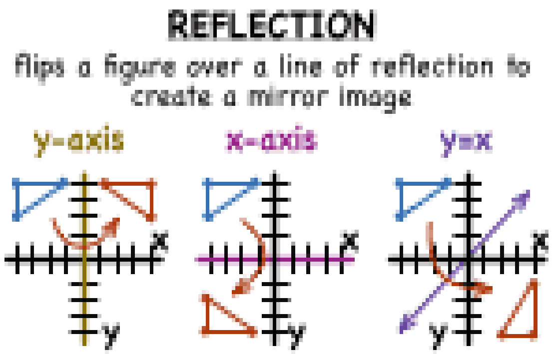
<table>
<tr><td colspan="2">Symmetry</td></tr>
<tr><td colspan="2">Lines of Symmetry</td></tr>
<tr><td>Points to Remember</td><td>Illustration/ Example</td></tr>
<tr><td>
A rhombus has 2 lines of symmetry

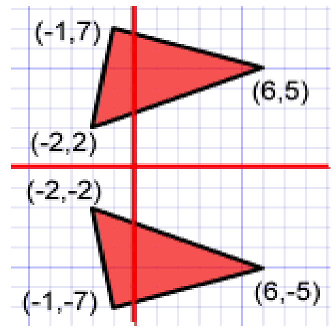
Non-example

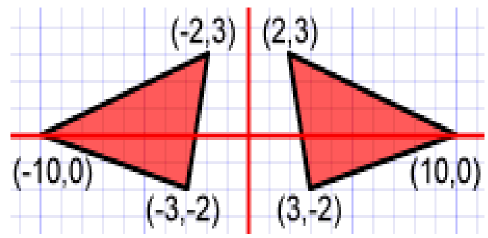
The scalene triangle does not have any lines of symmetry.</td><td>
A rectangle has 2 lines of symmetry

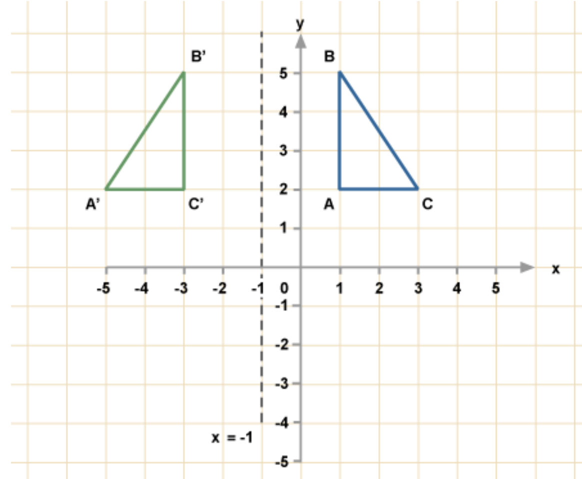

An equilateral triangle has 3 lines of symmetry</td></tr>
</table>

Transformations	
Translation	
Points to Remember	**Illustration/ Example**
* In a translation, all points in a line or object are changed in the same direction so there is no change in shape  	The points A(2,4), B(4,4), C(5,2), D(2,1) were translated under $\binom{7}{-3}$. Find the image A', B', C', D'  A' (-5, 1), B' (-3, 1), (-2, -1), (-5, -2)

<table>
<tr><td colspan="2">Transformations</td></tr>
<tr><td colspan="2">Reflection</td></tr>
<tr><td>Points to Remember</td><td>Illustration/ Example</td></tr>
<tr><td>A transformation is a change made to a figure
* The main types of transformations are: reflections, translations, rotations, and enlargements
* A reflection, or flip, is a transformation that creates a symmetry on the coordinate plane


* Matrices can be used to conduct transformations</td><td>Reflect the points (-1,7), (6,5), (-2,2) on the x-axis


Reflect the points (2,3), (10,0), (3,-2) on the y-axis


Reflect the points A(1,2), B(1,5) and C(3,2) on the line x=-1

</td></tr>
</table>

<table>
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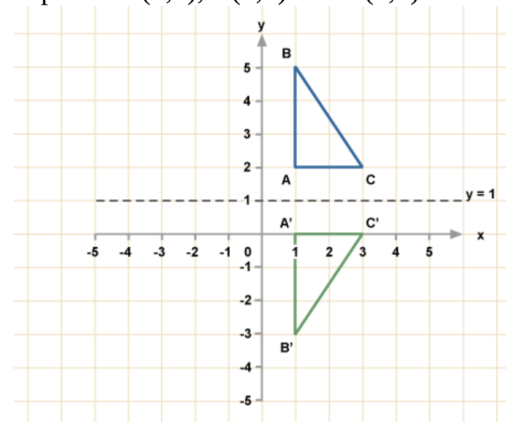
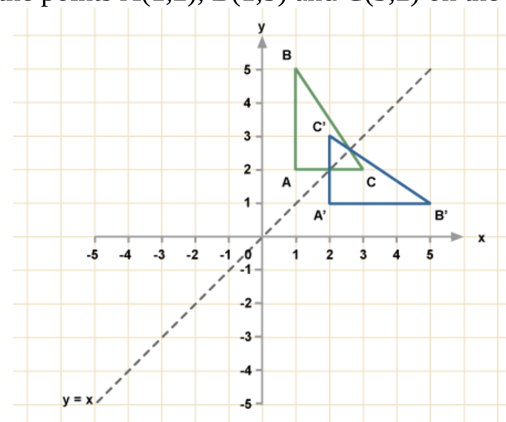
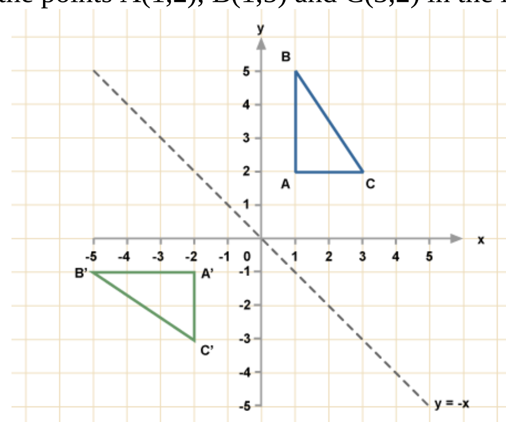
<tr><td colspan="2">Reflection</td></tr>
<tr><td>Points to Remember</td><td>Illustration/ Example</td></tr>
<tr><td></td><td>
Reflect the points A(1,2), B(1,5) and C(3,2) on the line y= 1



Reflect the points A(1,2), B(1,5) and C(3,2) on the line y=x



Reflect the points A(1,2), B(1,5) and C(3,2) in the line y= -x



</td></tr>
</table>

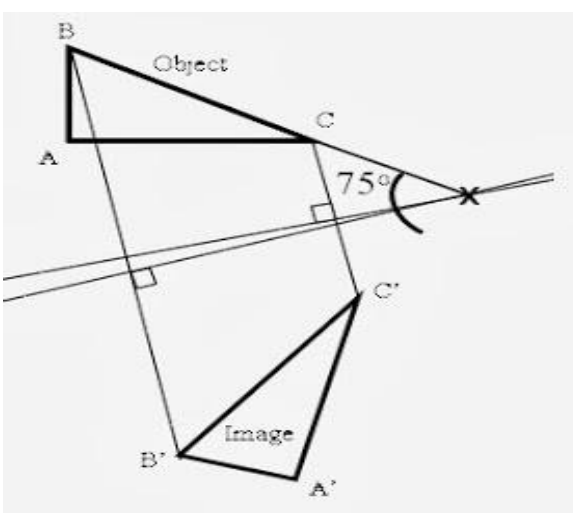
Transformations

Rotation

Points to Remember

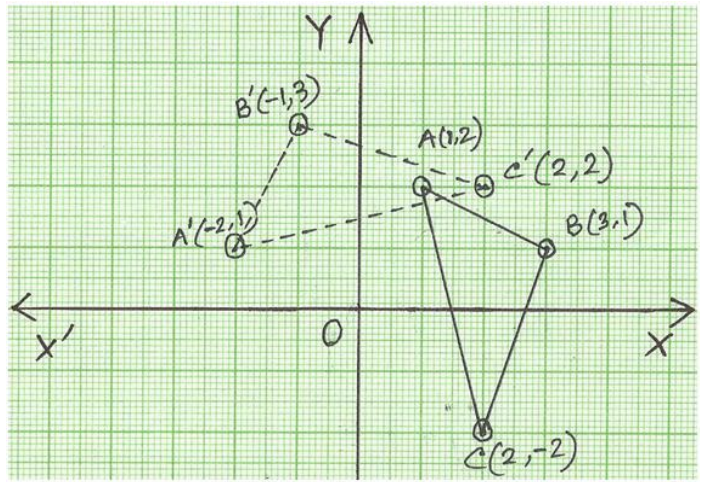
*A rotation is a transformation that turns a figure about a fixed point called a centre of rotation. A rotation has a centre and an angle. The angle is measured in an anticlockwise direction.

1. Pick a point B on the shape pre-transformation and locate the respective point post-transformation B'.
2. Draw line BB' .
3. Locate the midpoint M of B and B'.
4. Draw a perpendicular bisector (intersecting BB' at a right angle at M).
5. Repeat steps 1-4 for a second point C.
6. Extend the perpendicular bisectors (if necessary) so that they intersect.(Since perpendicular bisectors intersect the center of a circle, and since the circle containing B and B' and the circle containing C and C ' are both centered at the center of rotation), the intersection of the two perpendicular bisectors is the center of rotation.



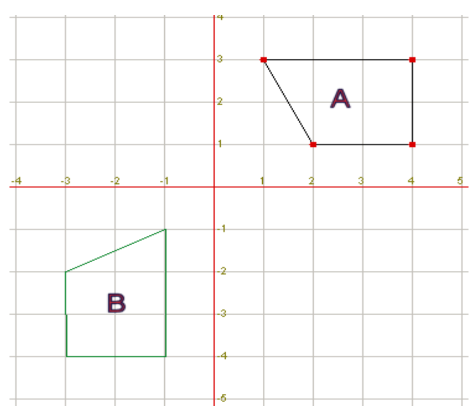
Illustration/ Example

Draw a triangle ABC on the graph paper. The co-ordinate of A, B and C being A (1, 2), B (3, 1) and C (2, -2), find the new position when the triangle is rotated through 90° anticlockwise about the origin



A (1, 2) will become A' (-2, 1)
B (3, 1) will become B' (-1, 3)
C (2, -2) will become C' (2, 2)
Thus, the new position of Δ ABC is Δ A'B'C'

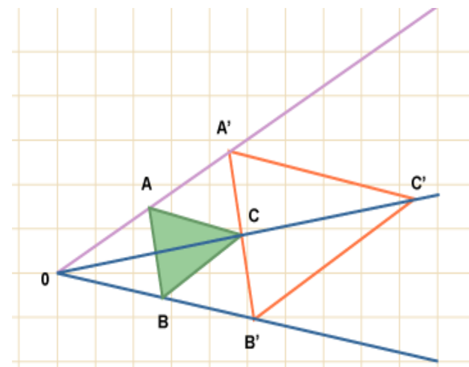
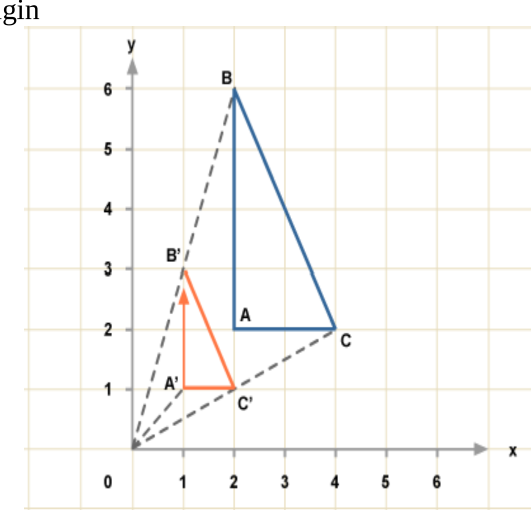
Describe fully the rotation with image shape A and object shape B.

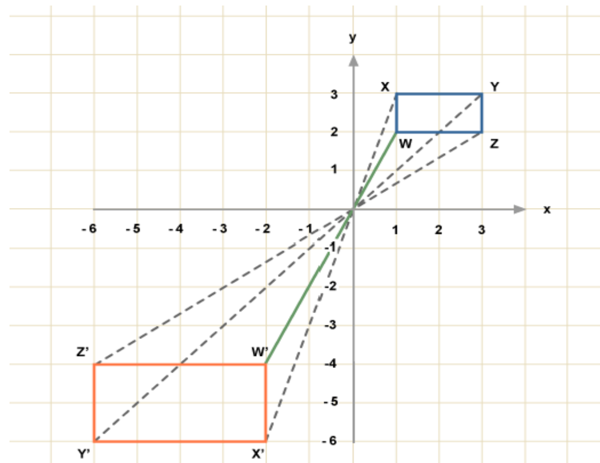


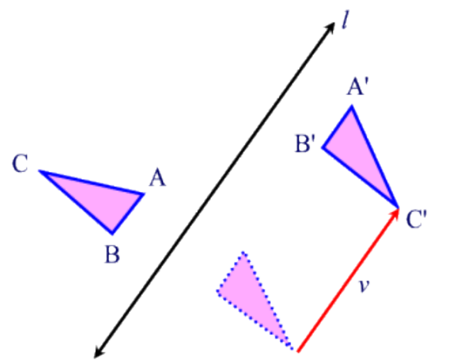
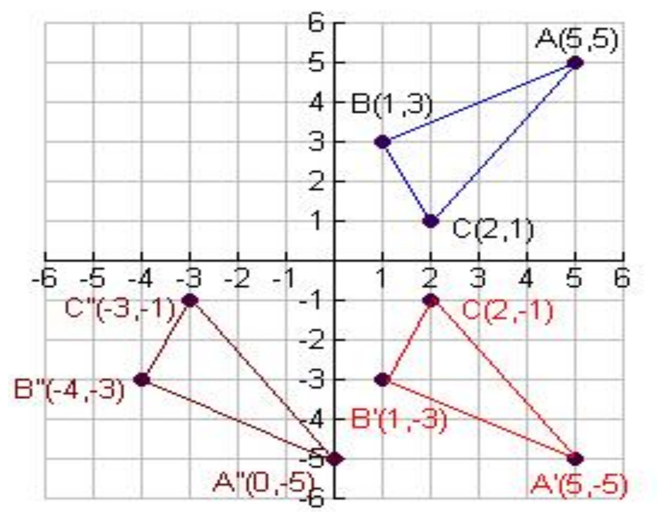
Solution: Rotation, 270°, anti-clockwise rotation , centre (-2, 2)

Transformations

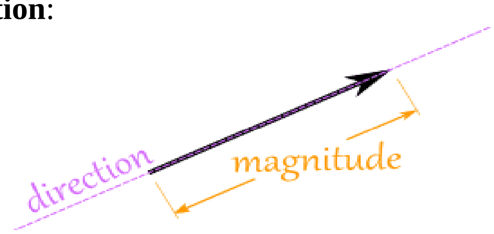
Enlargement

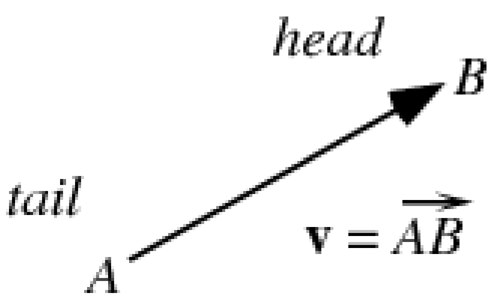
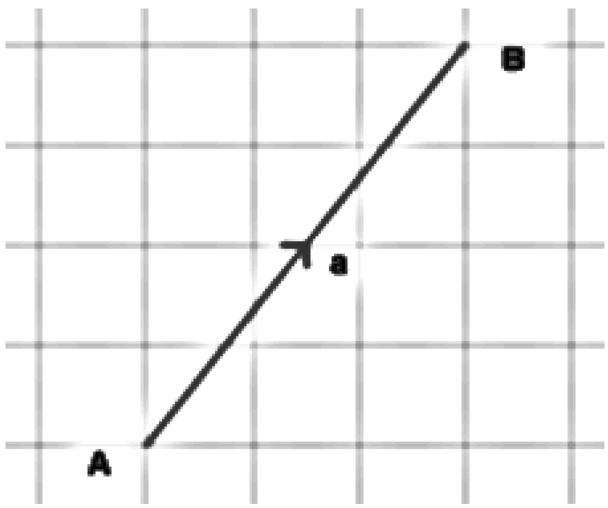
Points to Remember	Illustration/ Example
* An enlargement is a transformation that changes the size of a figure What is a scale factor? Enlarging a shape by a positive scale factor means changing the size of a shape by a scale factor from a particular point, which is called the centre of enlargement.	**Enlargement with scale factor greater than 1** Enlarge the object ABC by scale factor 2 
Fractional scale factors If we 'enlarge' a shape by a scale factor that is between -1 and 1, the image will be **smaller** than the object) If you enlarge it by a positive number greater than 1, the shape will get bigger.	**Enlargement with scale factor between -1 and +1** **Fractional Scale Factors** Enlarge triangle **ABC** with a scale factor $^{1}/_{2}$, centred about the origin  The scale factor is ½, so: OA' = ½OA, OB' = ½OB, OC' = ½OC Since the centre is the origin, we can in this case multiply each coordinate by ½to get the answers. A = (2, 2), so A' will be (1, 1). B = (2, 6), so B' will be (1, 3). C = (4, 2), so C' will be (2, 1).

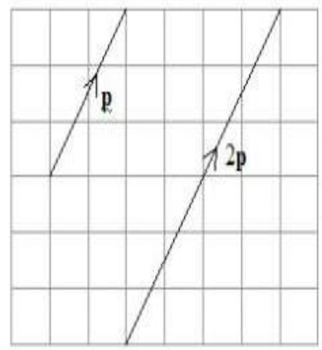
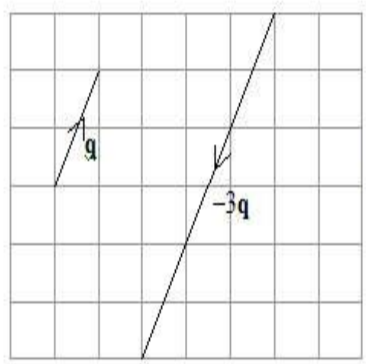
Transformations	
Enlargement	
Points to Remember	**Illustration/ Example**
Negative Scale Factors An enlargement using a negative scale factor is similar to an enlargement using a positive scale factor, but this time the image is on the other side of the centre of enlargement, and it is upside down to create you enlarged shape.	**Negative Scale Factors** Enlarge the rectangle **WXYZ** using a scale factor of - 2, centred about the origin.  The scale factor is -2, so multiply all the coordinates by -2. So OW' is 2OW. This time we extend the line WO beyond O, before plotting W'. In a similar way, we extend XO, YO and ZO and plot X',Y' and Z'. Can you see that the image has been turned upside down?

Transformations	
Glide-Reflection	
Points to Remember	**Illustration/ Example**
When a translation (a slide or glide) and a reflection are performed one after the other, a transformation called a **glide reflection** is produced. In a glide reflection, the line of reflection is parallel to the direction of the translation. It does not matter whether you glide first and then reflect, or reflect first and then glide. This transformation is commutative.  $\Delta A'B'C'$ is the image of ΔABC under a glide reflection that is a composition of a reflection over the line *l* and a translation through the vector *v*.	Examine the graph below. Is triangle A"B"C" a glide reflection of triangle ABC?  Answer: Yes, Triangle ABC is reflected on the x-axis to A'B'C' and then translated through 5 places to the left or $\begin{pmatrix}-5\\0\end{pmatrix}$

Vectors	
Scalar Quantities	
Points to Remember	**Illustration/ Example**
*Scalars are quantities that only have a magnitude, meaning they can be expressed with just a number. There are absolutely no directional components in a scalar quantity - only the magnitude of the medium e.g. Time - the measurement of years, months, weeks, days, hours, minutes, seconds, and even milliseconds; Volume - tons to ounces to grams, milliliters and micrograms Speed and - speed in miles or kilometers-per-hour, temperature	Give some real life examples of scalar quantities: Answer: Height of a building, time taken for a trip, temperature outside, an avocado on the scale reading 87.9 grams,

Vectors	
Vector Quantities	
Points to Remember	**Illustration/ Example**
A vector has **magnitude** (how long it is) and **direction**:  The length of the line shows its magnitude and the arrowhead points in the direction. e.g. Increase/Decrease in Temperature, Velocity	Give some real life examples of vector quantities: 10 meters to the left of the tree.

Vectors	
Vector Representation	
Points to Remember	**Illustration/ Example**
	Represent the diagram below in vector form:  This vector can be written as: $\overrightarrow{AB}$, **a**, or $\binom{3}{4}$

Vectors	
Product of a Vector and a Scalar	
Points to Remember	**Illustration/ Example**
* One method of multiplication of a vector is by using a scalar.	Calculate $3\begin{pmatrix}-2\\3\end{pmatrix} = \begin{pmatrix}-6\\9\end{pmatrix}$
Vectors	
Position Vector	
Points to Remember	**Illustration/ Example**
Describe vectors as 2x1 column vectors Associate position vectors with points Find the magnitude of a vector How do I use a column vector to describe a translation? How do I add or subtract vectors? Can any point be represented by a position vector? The length of a vector can be calculated using Pythagoras' theorem	Suppose we have a scalar 2 and a vector $\mathbf{p} = \begin{pmatrix}2\\3\end{pmatrix}$. Then $2\mathbf{p} = 2\times\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}4\\6\end{pmatrix}$. Note that in scalar multiplication, each component of the vector is multiplied by the scalar.  The magnitude $\lvert 2\mathbf{p}\rvert$ is twice as long as $\lvert\mathbf{p}\rvert$. Both vectors $2\mathbf{p}$ and $\mathbf{p}$ are in the same direction. If $\mathbf{q} = \begin{pmatrix}1\\2\end{pmatrix}$ then $-3\mathbf{q} = -3\times\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}-3\\-6\end{pmatrix}$  The magnitude $\lvert -3\mathbf{q}\rvert$ is three times as long as $\lvert\mathbf{q}\rvert$. Both vectors $-3\mathbf{q}$ and $\mathbf{q}$ are in opposite directions.

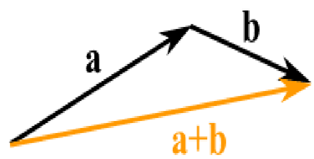
Vectors

Addition and Subtraction of Vectors

Points to Remember	Illustration/ Example

Points to Remember

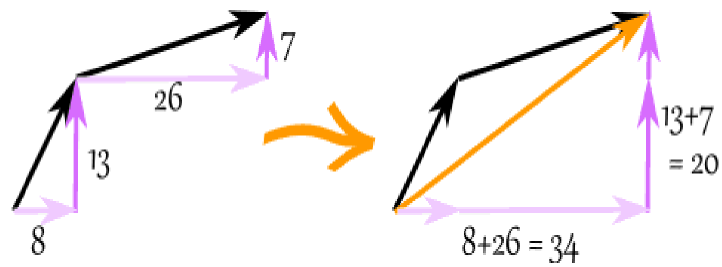
* Vector addition is simply the sum of the two vector's components. We can add two vectors by simply joining them head-to-tail:



Illustration/ Example

Add $\begin{pmatrix}8\\13\end{pmatrix}+\begin{pmatrix}26\\7\end{pmatrix}$ and illustrate this on a diagram

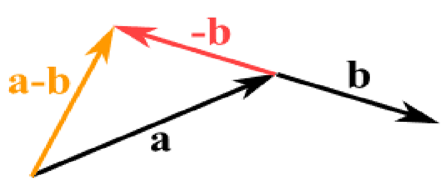
$\begin{pmatrix}8\\13\end{pmatrix}+\begin{pmatrix}26\\7\end{pmatrix}=\begin{pmatrix}34\\20\end{pmatrix}$



Points to Remember

We can also subtract one vector from another:

- first we reverse the direction of the vector we want to subtract,
- then add them as usual:

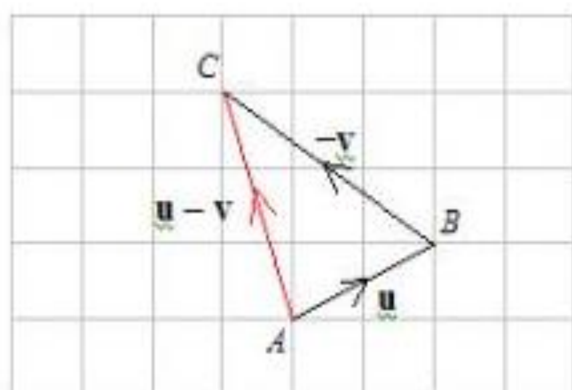


Illustration/ Example

If **u** = $\begin{pmatrix}2\\1\end{pmatrix}$ and **v** = $\begin{pmatrix}3\\-2\end{pmatrix}$

i) Illustrate **u – v** on a graph
ii) Calculate the value of **u – v**

i)



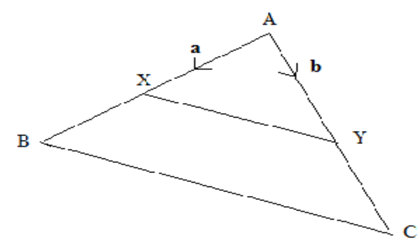
ii) **u – v** = $\begin{pmatrix}2\\1\end{pmatrix}-\begin{pmatrix}3\\-2\end{pmatrix}$

$= \begin{pmatrix}2\\1\end{pmatrix}+\begin{pmatrix}-3\\2\end{pmatrix}$

$= \begin{pmatrix}-1\\3\end{pmatrix}$

Vectors

Magnitude of a vector	
Points to Remember	**Illustration/ Example**
*The magnitude of a vector is shown by two vertical bars on either side of the vector: |a| We use Pythagoras' Theorem to calculate it: $\lvert\mathbf{a}\rvert = \sqrt{x^2 + y^2}$	1) What is the magnitude of the vector **b** = $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$ $\lvert\mathbf{b}\rvert = \sqrt{(6^2 + 8^2)} = \sqrt{(36+64)} = \sqrt{100} = 10$

Vectors	
Parallel Vectors	
Points to Remember	**Illustration/ Example**
One can use vectors to solve problems in Geometry e.g. to prove that two vectors are parallel. Two vectors are parallel if they have the same direction **<u>To prove that two vectors are parallel:</u>** If two vectors $\vec{u}$ and $\vec{v}$ are parallel, then one is a simple ratio of the other , or one is a multiple of the other $\vec{v} = k\vec{u}$	In the triangle ABC the points X and Y are the mid-points of AB and AC. Show that XY and BC are parallel.  $\overrightarrow{XY} = \overrightarrow{XA} + \overrightarrow{AY}$ $= -\mathbf{a} + \mathbf{b}$ $= \mathbf{b} - \mathbf{a}$ $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$ $= -2\mathbf{a} + 2\mathbf{b}$ $= 2\mathbf{b} - 2\mathbf{a}$ $= \mathbf{2\,(b - a)}$ This implies that one vector is a simple ratio of the other: $\frac{\overrightarrow{XY}}{\overrightarrow{BC}} = \frac{b-a}{2\,(b-a)} = \frac{1}{2}$ i.e. $\overrightarrow{XY} : \overrightarrow{BC} = 1{:}2$ OR one is a scalar multiple of the other (cross multiply) $\overrightarrow{BC} = 2\,\overrightarrow{XY}$ or $\overrightarrow{XY} = \frac{1}{2}\overrightarrow{BC}$ Hence, XY is parallel to BC and half its length.

Vectors	
Collinear Vectors	
Points to Remember	**Illustration/ Example**
Points that lie on the same line are called **collinear** points. **<u>To prove that two vectors are collinear:</u>** If two vectors are collinear, then one is a simple ratio of the other, or one is a multiple of the other $\vec{v} = k\vec{u}$ **and** they have a common point.	The position vectors of points P, Q and R are vectors a + b, 4a - b and 10a – 5b respectively. Prove that P, Q and R are collinear. $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$ = (-**a** –**b**) + (4**a** – **b**) = 3**a** – 2**b** $\overrightarrow{QR} = \overrightarrow{QO} + \overrightarrow{OR}$ = (-4a + b) + (10a – 5b) = 6a – 4b = 2 (3**a** – 2**b**) This implies that one vector is a simple ratio of the other **and they have a common point Q** $\frac{\overrightarrow{PQ}}{\overrightarrow{QR}} = \frac{3a-2b}{2\,(3a-2b)} = \frac{1}{2}$ i.e. $\overrightarrow{PQ}$: $\overrightarrow{QR}$ = 1:2 **OR** one is a scalar multiple of the other (cross multiply) $\overrightarrow{QR} = 2\,\overrightarrow{PQ}$ or $\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{QR}$ Since $\overrightarrow{QR} = 2\,\overrightarrow{PQ}$ **and they have a common point Q**, then P, Q and R are collinear.

Matrices

Introduction to Matrices

Points to Remember

* A matrix is an ordered set of numbers listed in rectangular form and enclosed in curved brackets. It is usual to denote matrices in capital letters
* In defining the ORDER of a matrix, the number of rows is always stated first and then the number of columns.

* There are different types of matrices such as square matrices, diagonal matrices and identity matrices.

Row Matrix- A row matrix is formed by a single row e.g. (a b c)

Column Matrix- A column matrix is formed by a single column e.g. $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

Rectangular Matrix- A rectangular matrix is formed by a different number of rows and columns, and its dimension is noted as: **mxn** **e.g.** $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ is 3x2

Square Matrix - A square matrix is formed by the same number of rows and columns e.g $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is 2x2

Diagonal Matrix- In a diagonal matrix, all the elements above and below the diagonal are zeroes e.g.

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

Identity Matrix-An identity matrix is a diagonal matrix in which the diagonal elements are equal to 1

e.g. $$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Singular matrix- see topic on **Inverse Singular** below

A zero or null matrix is a matrix with 0 as the element for all its cells (rows and columns).

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Illustration/ Example

Examples

1) $(2 \quad 9 \quad -3)$ is a 1 x 3 **row** matrix

2) $\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$ is a 3 x 1 **column** matrix

3) $\begin{pmatrix} 5 & 7 & 9 \\ 3 & 2 & 5 \end{pmatrix}$ is a 2x3 **rectangular** matrix

4) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ is a 2 x 2 **square** matrix

5) $\begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 8 \end{pmatrix}$ is a **diagonal** Matrix

6) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is an **identity** matrix

7) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ is the **zero or null** matrix

Matrices	
Addition and Subtraction of Matrices	
Points to Remember	**Illustration/ Example**
Two matrices may be added or subtracted provided they are of the SAME ORDER. Addition is done by adding the corresponding elements of each of the two matrices. $\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$	Examples: 1) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$ 2) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} -4 & -4 \\ -4 & -4 \end{pmatrix}$

Matrices	
Multiplication of Matrices	
Points to Remember	**Illustration/ Example**
Multiplication is only possible if the row vector and the column vector have the same number of elements. To multiply the row by the column, one multiplies corresponding elements, then adds the results $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} I \\ S \end{pmatrix} = \begin{pmatrix} aI+bS \\ cI+dS \end{pmatrix},$ Also, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} (ae+bg) & (af+bh) \\ (ce+dg) & (cf+dh) \end{pmatrix}$	Examples: 1) $2\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 8 & 4 \end{pmatrix}$ 2) $(1\ \ 2\ \ 3)\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = (1\times 4) + (2\times 5) + (3\times 6) = (22)$ A 1×3 matrix multiplied by a 3×1matrix gives a 1×1 matrix 3) $\begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 4 & -1 \end{pmatrix}$ $= \begin{pmatrix} (2\times -2+1\times 4) & (2\times 3+1\times -1) \\ (3\times -2+5\times 4) & (3\times 3+5\times -1) \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ 14 & 4 \end{pmatrix}$ A 2×2 matrix multiplied by a 2×2 matrix gives a 2×2 matrix

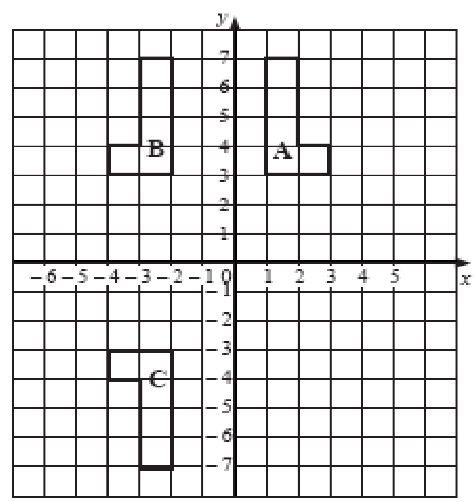
Matrices	
Inverse of a Matrix	
Points to Remember	**Illustration/ Example**
If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Then the inverse is $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ and the determinant is $\det A = \lvert A \rvert = ad - bc$	If $A = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$, find A^{-1} $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ $= \frac{1}{(3)(2)-(1x4)}\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}$ $= \begin{pmatrix} 1 & -1/2 \\ -2 & 3/2 \end{pmatrix}$

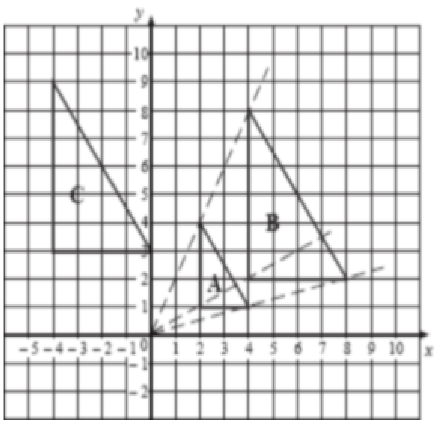
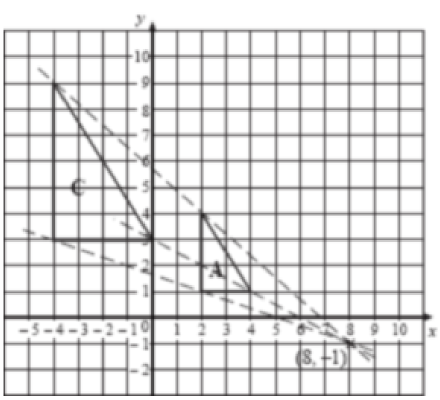
Matrices	
Singular Matrix	
Points to Remember	**Illustration/ Example**
A singular matrix is a square matrix that has no inverse A matrix is singular if and only if its determinant is zero i.e. ad – bc = 0 If the determinant of a matrix is 0, the matrix has no inverse	Determine if the matrix A = $\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$ is singular Det A = ad- bc = (2)(3) – (6)(1) = 6 – 6 = 0

Matrices	
Simultaneous Equations	
Points to Remember	**Illustration/ Example**
One of the most important applications of matrices is to the solution of linear simultaneous equations	Solve the simultaneous equation using a matrix method x + 2y = 4 3x – 5y = 1 This can be written in matrix form AX = B: $\begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{(1x{-}5)-(2x3)} \begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $= \frac{1}{(1x{-}5)-(2x3)} \begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $= \frac{1}{(-5)-(6)} \begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $= \frac{-1}{11} \begin{pmatrix} -22 \\ -11 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Matrices	
Transformational matrices	
Points to Remember	**Illustration/ Example**
R= 90° rotation about the origin, given the matrix. This transformation matrix rotates the point matrix 90 degrees anti-clockwise. When multiplying by this matrix, the point matrix is rotated 90 degrees anti-clockwise around (0,0). $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$	R = 90° anti-clockwise rotation about the origin $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}(0\ x\ 4) + & (-1\ x\ 3)\\(1\ x\ 4) + & (0\ x\ 3)\end{pmatrix} = \begin{pmatrix}-3\\4\end{pmatrix}$
S= 180° anticlockwise rotation about the origin, given the matrix. This transformation matrix creates a rotation of 180 degrees. When multiplying by this matrix, the point matrix is rotated 180 degrees around (0,0). This changes the sign of both the x and y co-ordinates. $\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$	S = 180° anti-clockwise rotation about the origin $\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}(-4 \times 1) + & (3 \times 0)\\(4 \times 0) + & (3 \times -1)\end{pmatrix} = \begin{pmatrix}-4\\-3\end{pmatrix}$
T= 270° rotation about the origin, given the matrix $\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$	T = 270° anti-clockwise rotation about the origin $\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}(4 \times 0) + & (3 \times 1)\\(4\ \times -1) + & (3 \times 0)\end{pmatrix} = \begin{pmatrix}3\\-4\end{pmatrix}$
I= 360° rotation about the origin, given the matrix. This transformation matrix is the identity matrix. When multiplying by this matrix, the point matrix is unaffected and the new matrix is exactly the same as the point matrix $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$	I = Identity Matrix $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}(4 \times 1) + & (3 \times 0)\\(4 \times 0) + & (3 \times 1)\end{pmatrix} = \begin{pmatrix}4\\3\end{pmatrix}$
X= reflection on x axis, given the matrix. This transformation matrix creates a reflection in the x-axis. When multiplying by this matrix, the x co-ordinate remains unchanged, but the y co-ordinate changes sign $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	X = Reflection on x axis $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}(4 \times 1) + & (3 \times 0)\\(4 \times 0) + & (3 \times -1)\end{pmatrix} = \begin{pmatrix}4\\-3\end{pmatrix}$
Y= reflection on y axis, given the matrix This transformation matrix creates a reflection in the y-axis. When multiplying by this matrix, the y co-ordinate remains unchanged, but the x co-ordinate changes sign $\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$	Y = Reflection on y axis $\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}(4 \times -1)\ + & (3 \times 0)\\(4\ \times\ 0)\ + & (3 \times 1)\end{pmatrix} = \begin{pmatrix}-4\\3\end{pmatrix}$
W= reflection on y= x, given the matrix. This transformation matrix creates a reflection in the line	W= reflection on y = x

Matrices	
Transformational matrices	
Points to Remember	**Illustration/ Example**
y=x. When multiplying by this matrix, the x co-ordinate becomes the y co-ordinate and the y-ordinate becomes the x co-ordinate. $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ Reflection on y = - x, given the matrix. This transformation matrix creates a reflection in the line y=-x. When multiplying by this matrix, the point matrix is reflected in the line y=-x changing the signs of both co-ordinates and swapping their values. $\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}(4\times 0)+ & (1\times 3)\\(1\times 4)+ & (0\times 3)\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix}$ Reflection on y = - x, $\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}\begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}(4\times 0) & + & (-1\times 3)\\(-1\times 4)+ & & (0\times 3)\end{pmatrix} = \begin{pmatrix}-3\\-4\end{pmatrix}$

Matrices	
Combining Transformations	
Points to Remember	**Illustration/ Example**
We see that sometimes 2 transformations are equivalent to a single transformation.	1) The diagram shows how the original shape A is first reflected to B, and B is then reflected to C.  Solution: A rotation of 180° about the origin would take A straight to C.

Matrices	
Combining Transformations	
Points to Remember	**Illustration/ Example**
	A triangle is to be enlarged with scale factor 2, using the origin as the centre of enlargement. Its image is then to be translated along the vector $\begin{pmatrix} -8 \\ 1 \end{pmatrix}$ The coordinates of the corners of the triangle are (2, 1), (2, 4) and (4, 1). What ***single*** transformation would have the same result? The diagram shows the original triangle, A; the enlargement takes it to B, which is then translated to C. The triangle A could be enlarged with scale factor 2 to give C.  This diagram shows that the centre of enlargement would be the point (8, –1). The single transformation that will move triangle A to triangle C is an enlargement, scale factor 2, centre (8, –1). 

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